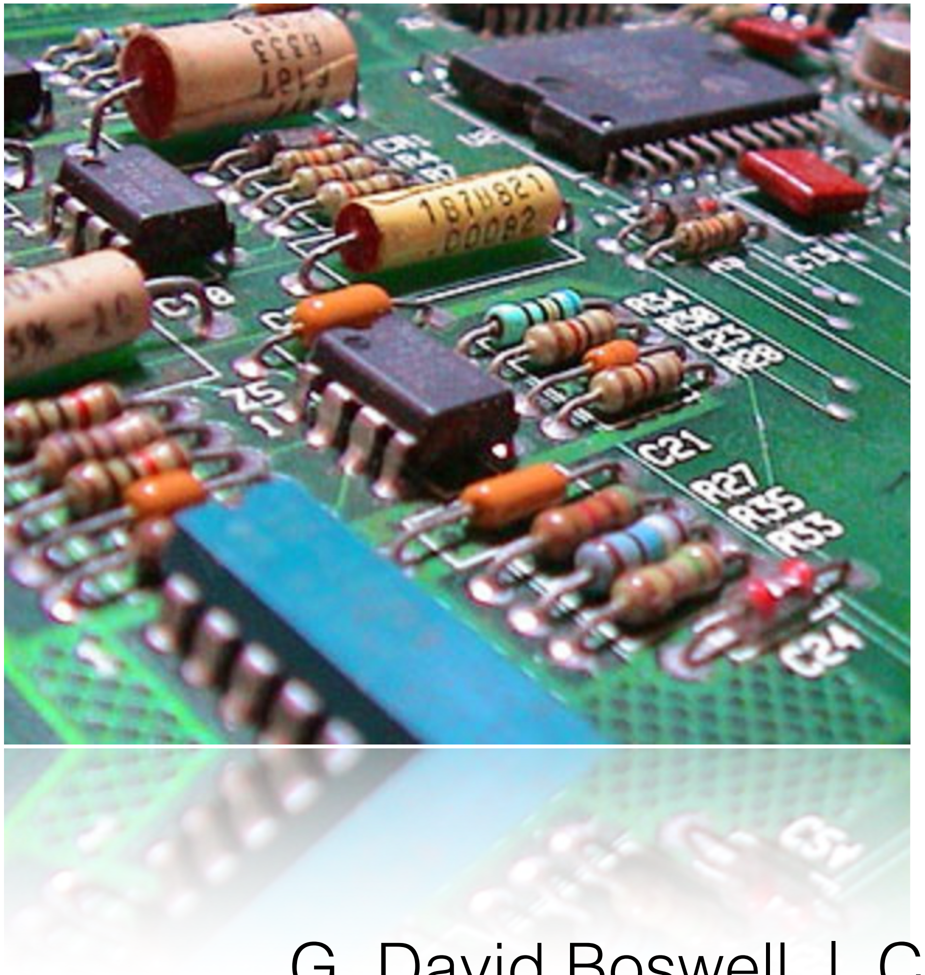
# Electric Circuit Analysis



## TUTORIAL CONTENT

1. The Anatomy of the Cell
2. Network Reductions
3. p.d. and Current Dividers
4. Ohm's, Joule's and intro to Kirchhoff's KCL & KVL*

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**The 21st Century**

# Anatomy of the Cell

## Example 01.

The e.m.f. of a cell is **1.67 V**. When it is connected to a **10-Ω** resistor, its terminal p.d. was measured to be **1.50V**.

Calculate

(a) the current through the **10-Ω** external resistor,

(b) the internal resistance of the cell, and

(c) the rate of energy loss in the cell (i.e., power loss, $P_{loss}$ ).

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# Anatomy of the Cell

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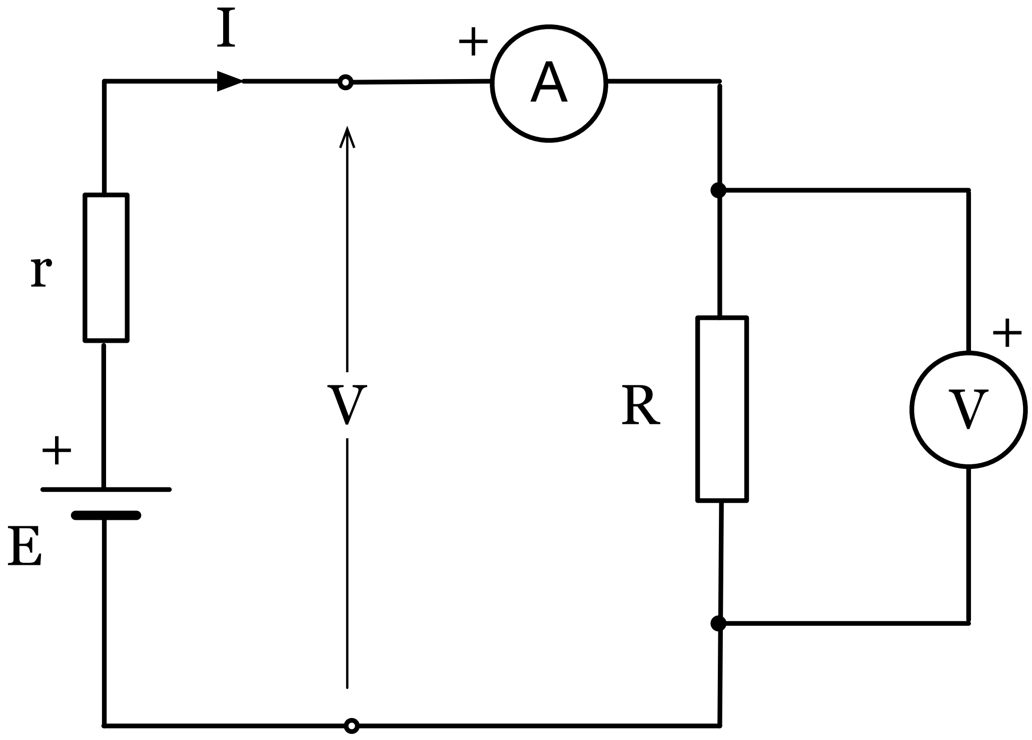
## Response 01.

First, whenever relevant, draw a simplified line diagram that shows problem understanding.

Note very well that when the cell <u>supplies</u> current **I** to the load **R**, then the e.m.f., **E**, will be greater than the terminal p.d., **V**. In this case, $\mathbf{E - V = Ir}$



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# Anatomy of the Cell

Terminal p.d.:

$V = IR$

## Response 01(a).

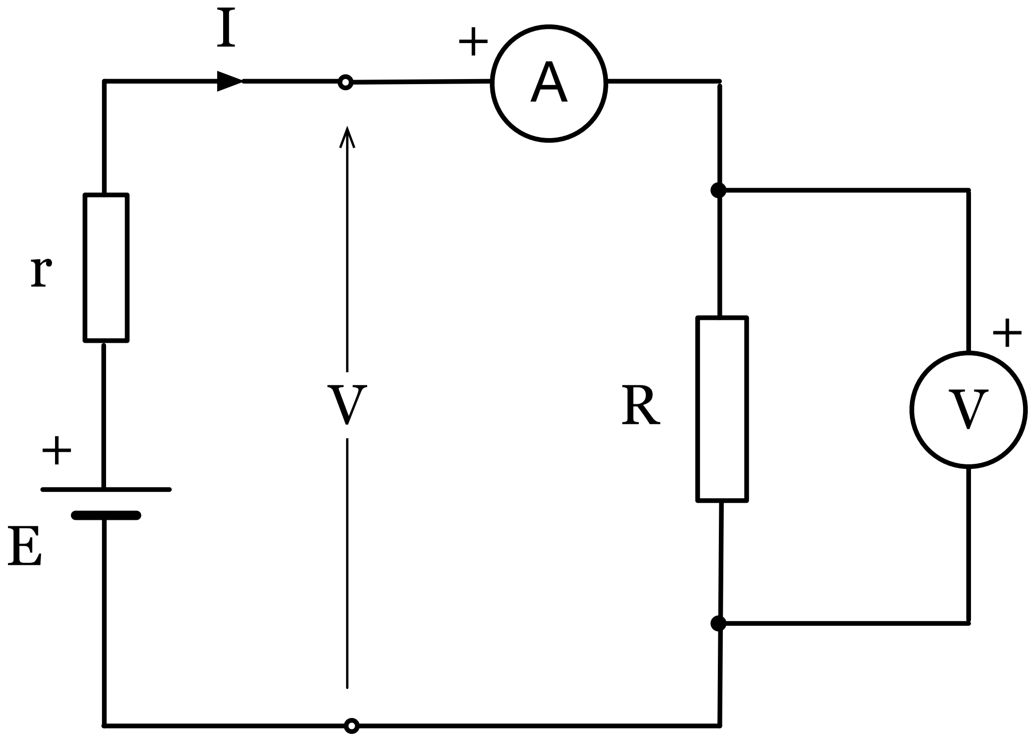
*The terminal p.d. is also the voltage across the load resistor, R,*

Applying Ohm's Law,

$$I = \frac{V}{R}$$

$$= \frac{1.50\ V}{10\ \Omega}$$

$$= 0.15\ A$$



# Anatomy of the Cell

Cell Model:

$$V = E - Ir$$

## Response 01(b).

Recall that, for a *source*,

$$E - V = Ir$$

Therefore,

$$r = \frac{E - V}{I}$$

$$= \frac{(1.67 - 1.50)\ V}{0.15\ A}$$

$$= 1.13\ \Omega$$

$$\approx 1.1\ \Omega$$

## Response 01(c).

Due to the internal resistance,

$$P_{loss} = I^2 r$$

$$= (0.15\ A)^2 \times 1.13\ \Omega$$

$$\approx 26\ \text{mW}$$

**Reader** ~ Another approach is:

$$P_{loss} = P_{in} - P_{out}$$

$$= EI - VI$$

$$= (E - V)I$$

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# Anatomy of the Cell

## Example 02.

The terminal p.d. of an interconnections of cells (i.e. a battery) on open-circuit is $\mathbf{100\ V}$. When it delivers a current of $\mathbf{20.0 A}$ to a circuit, its terminal p.d. falls to $\mathbf{95.4V}$.

Calculate

(a) the efficiency of the battery, $\eta$.

(b) the power output of the battery, $P_{out}$ .

# Anatomy of the Cell

Efficiency:

$$\eta = \frac{V}{E} = \frac{R}{R+r}$$

## Response 02(a).

If $\mathbf{P_{in}}$ is the input power conversion to electrical inside the battery, and $\mathbf{P_{out}}$ is the output power conversion at the terminals of the battery, then the efficiency of the battery is

$$\eta = \frac{P_{out}}{P_{in}} \times 100\%$$

$$= \frac{V}{E} \times 100\% \qquad \text{since } P_{out} = VI \;\text{ and } P_{in} = EI$$

$$= \frac{95.4}{100} \times 100\%$$

$$= 95.4\ \%$$

# Anatomy of the Cell

Output Power

$P_{out} = VI = \eta EI$

## Response 02(b).

$\mathbf{P_{out}}$ is the output power conversion from electrical to other forms realized <u>at the terminals</u> of the battery.

For this problem,

$$\begin{aligned} P_{out} &= VI \\ &= 95.4\ V \times 20.0\ A \\ &= 1{,}890\ W \\ &= 1.89\ \mathrm{kW} \end{aligned}$$

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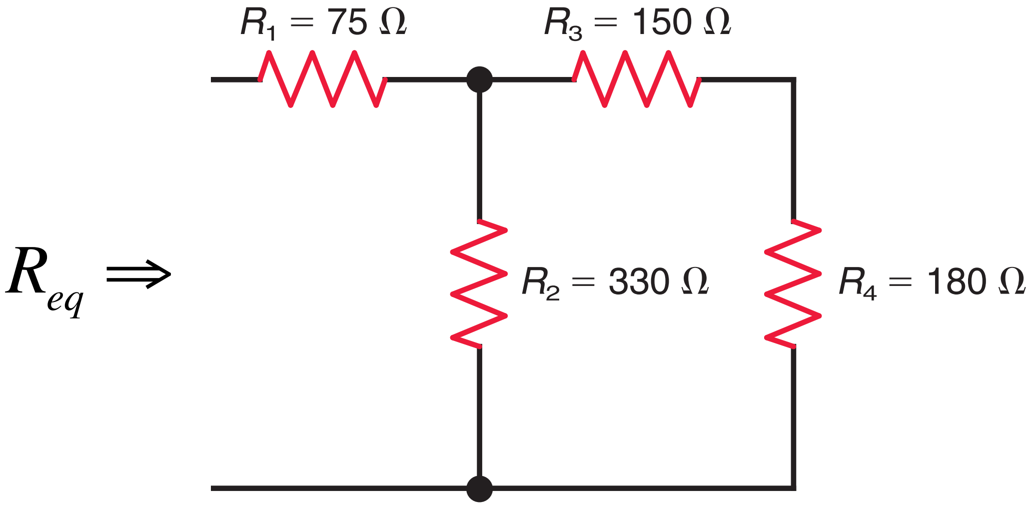
# Network Reduction

## Example 03.

(a) Find $R_{eq}$, the equivalent resistance of the network shown.

(b) Repeat (a) when. $R_4 = 0\Omega$ (i.e., an ideal short circuit)

(c) Repeat (a) when $R_4 \to \infty$ (i.e., a practical open circuit)



# Network Reduction

Series:

$$R_S = \sum_{i=1}^{n} R_i$$

## Response 03(a).

Using the formulations for resistors in series and parallel,

$$R_{eq} = R_1 + \frac{R_2\left(R_3 + R_4\right)}{R_2 + \left(R_3 + R_4\right)} \qquad \text{"product over sum"}$$

$$= R_1 + \frac{R_2}{2} \qquad \text{since, in this case, } R_2 = R_3 + R_4$$

$$= 75\Omega + 165\Omega$$

$$= 240\Omega$$

---

NB: (a) For n equal resistors in series: $R_S = nR$

(b) For n equal resistors in parallel: $R_P = \frac{R}{n}$

# Network Reduction

Parallel:

$$R_P = \left(\sum_{i=1}^{n} R_i\right)^{-1}$$

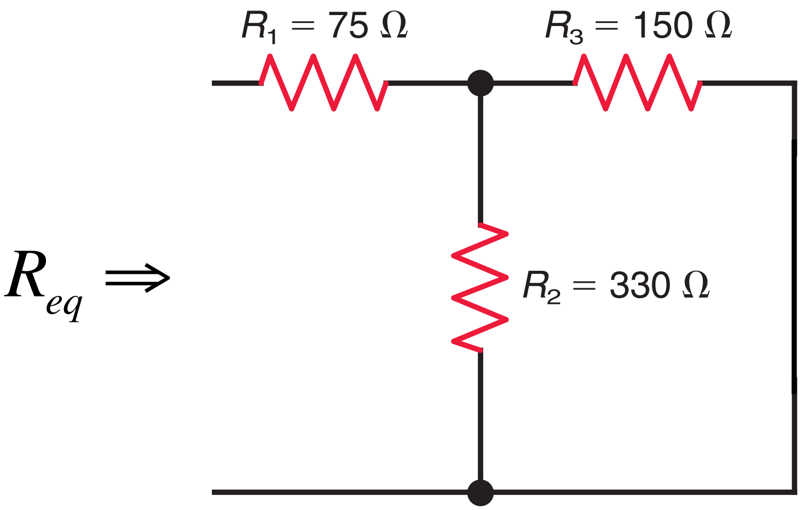
## Response 03(b).

When **R₄** is "shorted," then

$$R_{eq} = R_1 + \frac{R_2 R_3}{R_2 + R_3}$$

$$= 75\Omega + \left(\frac{330 \times 150}{330 + 150}\right)\Omega$$

$$= 178.1\Omega$$



NB: For 2 resistors in parallel, we utilize

"product over sum": $R_a // R_b = \dfrac{R_a R_b}{R_a + R_b}$

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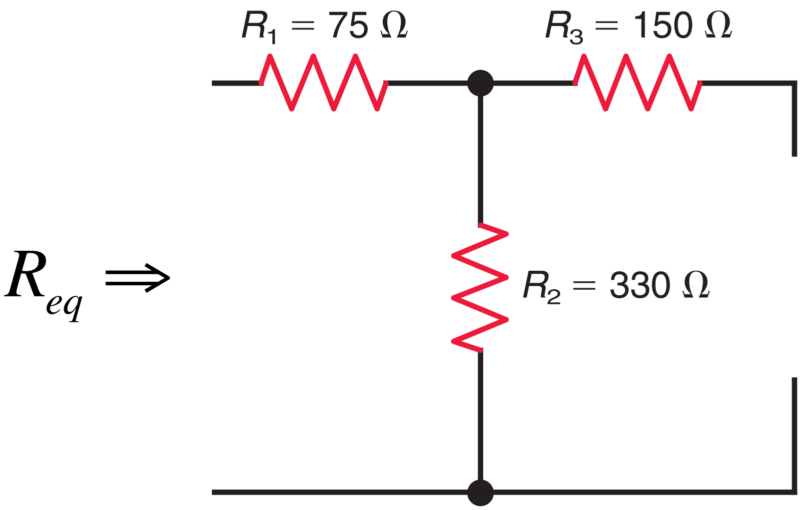
# Network Reduction

## Response 03(c).

When $R_4$ is "opened," then because its loop is broken, $R_1$ and $R_2$ are now in series such that

$$R_{eq} = R_1 + R_2$$
$$= 75\Omega + 330\Omega$$
$$= 405\Omega$$

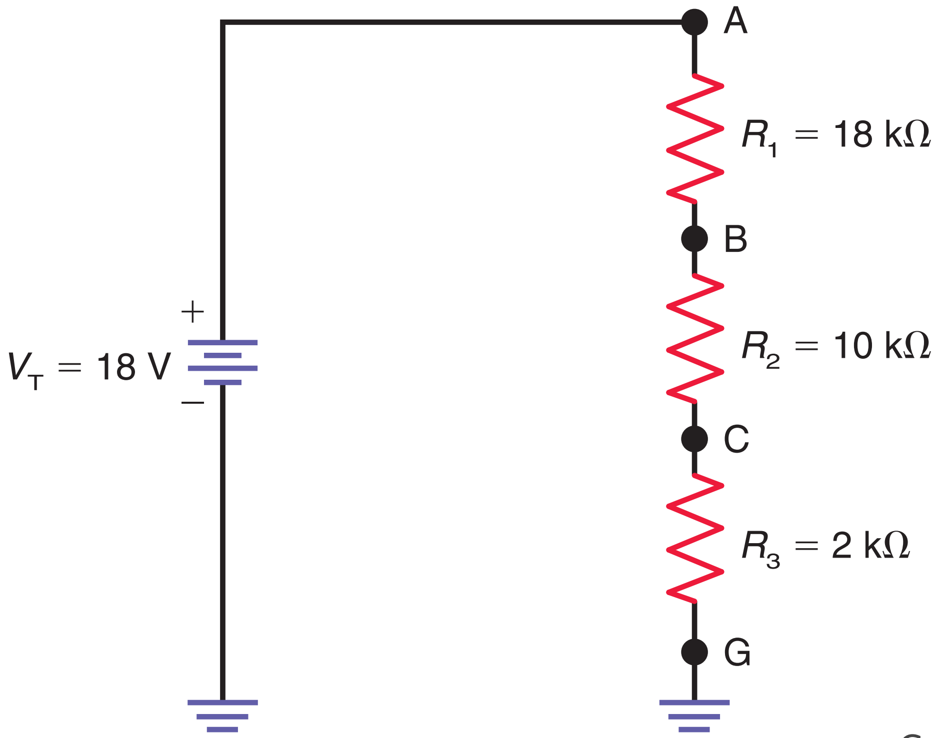


# Potential Divider

## Example 04.

(a) Find $R_T$, the total resistance of the network shown.

(b) Compute the p.d. across each resistor.



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# Potential Divider

## Response 04(a).

Using the formulation for

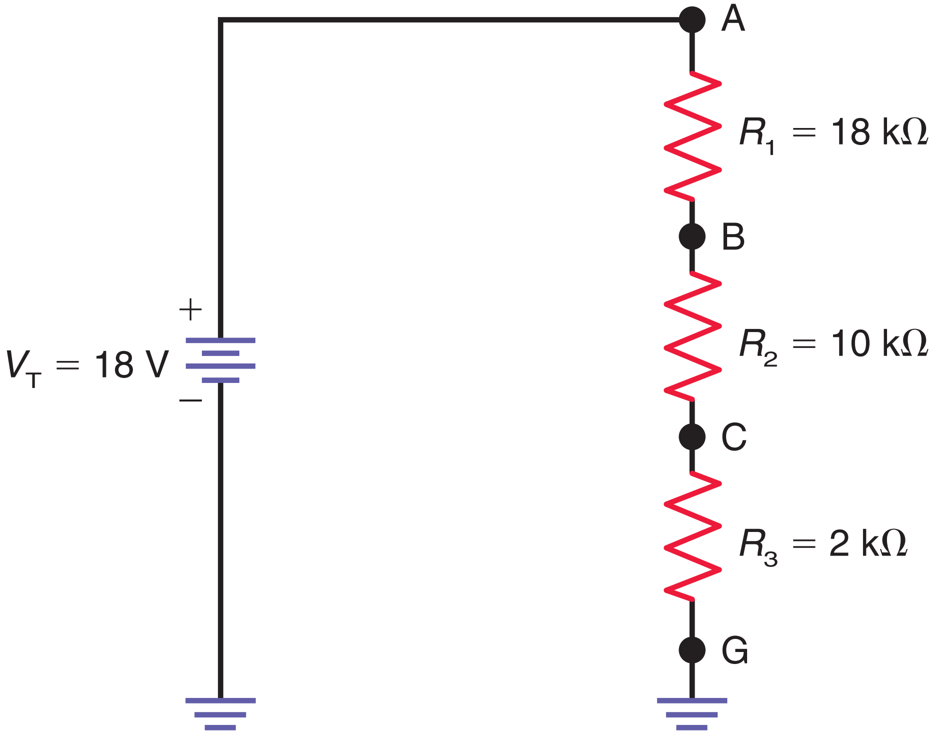
combining 3 resistors in series,

$$R_T = \sum_{i=1}^{3} R_i$$

$$= R_1 + R_2 + R_3$$

$$= 18\Omega + 10\Omega + 2\Omega$$

$$= 30\ \Omega$$



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# Potential Divider

$$V_i = \frac{R_i}{R_S} V$$

## Response 04(b).

The p.d. across each resistor in the series network is

$$V_1 = \frac{R_1}{R_T} V_T = \frac{18\Omega}{30\Omega} \times 18V = 10.8\ \mathrm{V}$$

$$V_2 = \frac{R_2}{R_T} V_T = \frac{10\Omega}{30\Omega} \times 18V = 6.0\ V$$

$$V_3 = \frac{R_3}{R_T} V_T = \frac{2\Omega}{30\Omega} \times 18V = 1.2\ V$$

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Note that, in accordance with KVL: $V_T = \sum_{i=1}^{n} V_i$

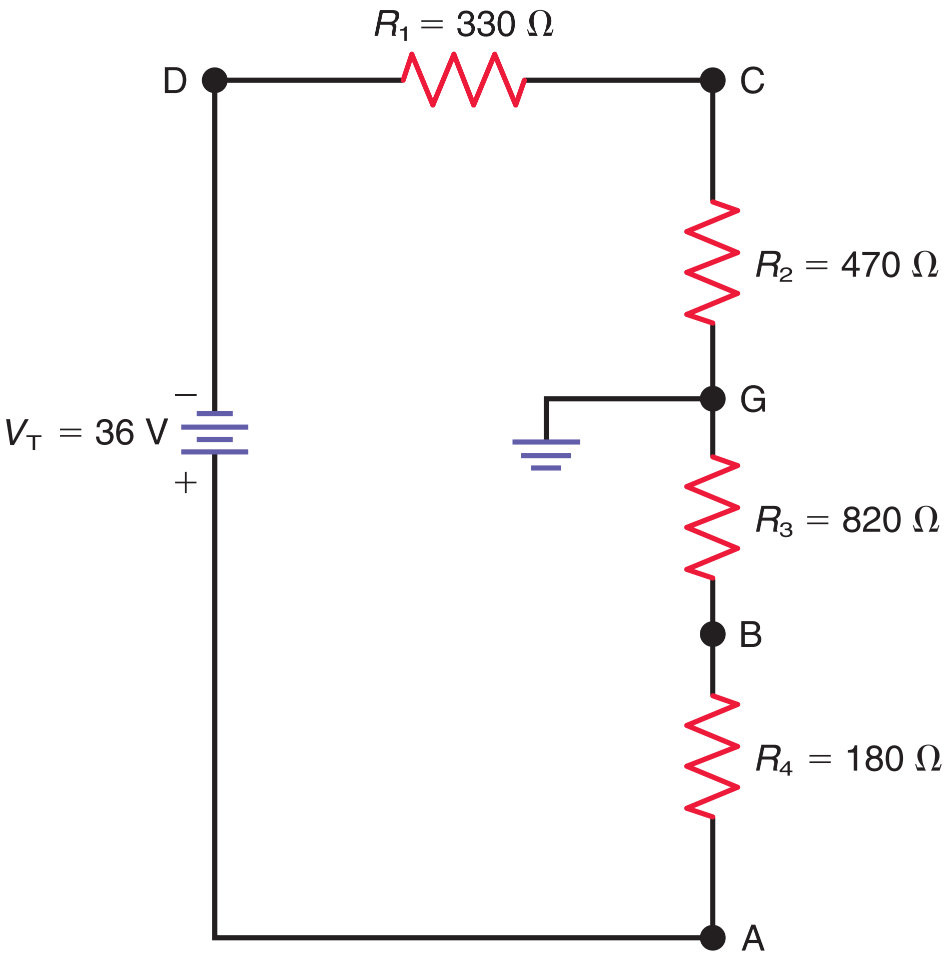
# Potential Divider

## Example 05.

(a) Find $R_T$ , the total resistance of the network shown.

(b) Compute the p.d. across each resistor.

(c) Hence, calculate the $\mathbf{V_C}$ and $\mathbf{V_B}$, the potentials at nodes **C** and **B**, respectively.



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# Potential Divider

## Response 05(a).

Using the formulation for combining 4 resistors in series, we get

$$
\begin{aligned}
R_T &= \sum_{i=1}^{4} R_i \\
&= R_1 + R_2 + \ldots + R_4 \\
&= 330 + 470 + 820 + 180 \quad \Omega \\
&= 1{,}800\ \Omega \\
&= 1.80\ \mathrm{k\Omega}
\end{aligned}
$$

# Potential Divider

## Response 05(b).

The respective p.d. across the first 2 resistors in the network is

$$V_1 = \frac{R_1}{R_T} V_T = \frac{330\Omega}{1{,}800\Omega} \times 36V = 6.6\ \mathrm{V}$$

$$V_2 = \frac{R_2}{R_T} V_T = \frac{470\Omega}{1{,}800\Omega} \times 36V = 9.4\ V$$

# Potential Divider

## Response 05(b). (cont.d)

Similarly, for the other 2 resistors in the series circuit,

$$V_3 = \frac{R_3}{R_T} V_T = \frac{820\Omega}{1{,}800\Omega} \times 36V = 16.4\ V$$

$$V_4 = \frac{R_4}{R_T} V_T = \frac{180\Omega}{1{,}800\Omega} \times 36V = 3.6\ V$$

---

Note that, in accordance with KVL: $V_T - \sum_{i=1}^{n} V_i = 0 \text{ volts}$

# Potential Divider

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## Response 05(c).

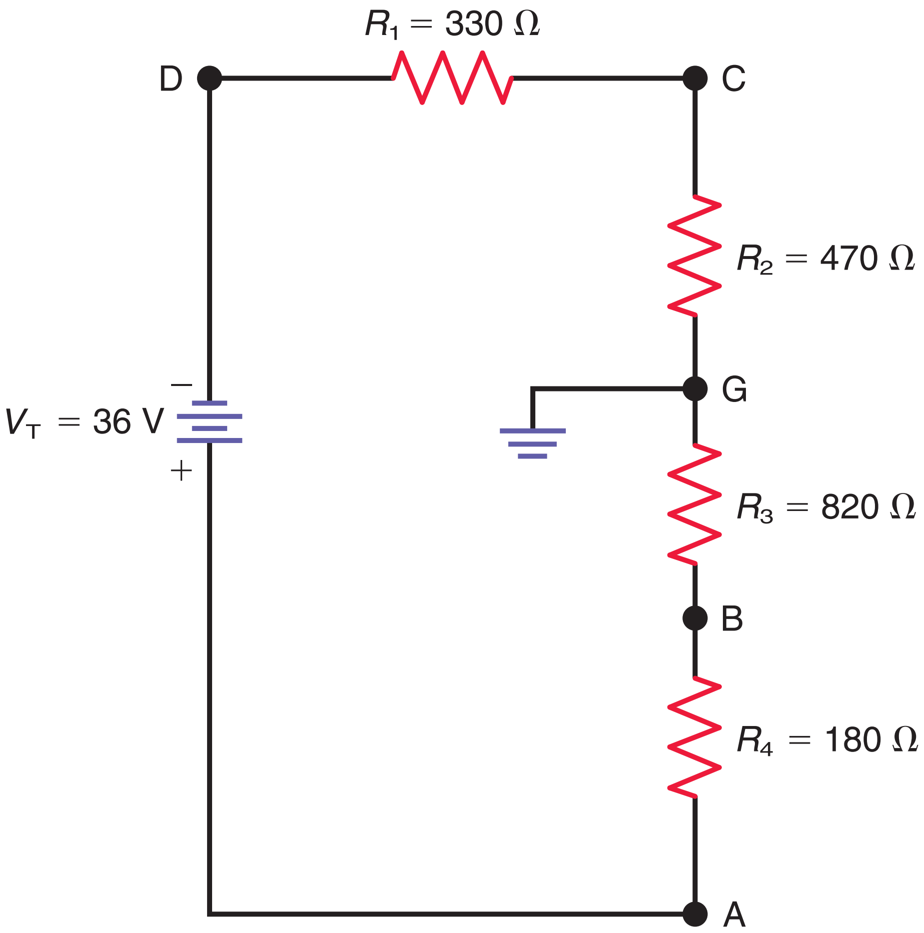
The reference p.d. is $V_G = 0V$.

And, based on battery polarity,

$$V_C = V_G - V_2 = 0V - 9.4V = -9.4V$$

$$V_B = V_G + V_3 = 0V + 16.4V = 16.4V$$

*The lowest voltage potential is at node D and the highest voltage potential is at node A.*



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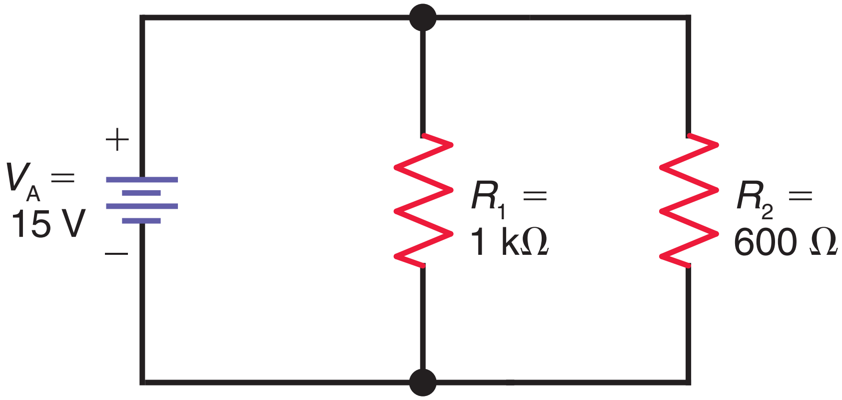
# Current Divider

## Example 06.

(a) Find $R_T$, the total resistance of the network shown.

(b) Calculate the total current in the circuit.

(c) Compute the current through each resistor and verify that Kirchhoff's Current Law (KCL) holds true.



# Current Divider

## Response 06(a).

Using the formulation for combining 2 resistors in parallel, we get

$$R_T = \left( \frac{1}{R_1} + \frac{1}{R_{12}} \right)^{-1}$$

$$\equiv \frac{R_1 R_2}{R_1 + R_2}$$

$$= \frac{1000 \times 600}{1000 + 600} \ \Omega$$

(all values in the same units!)

$$= 375 \ \Omega$$

# Current Divider

## **Response 06(b).**

Using Ohm's Law, the total current in the circuit is

$$I_T = \frac{V_A}{R_T}$$

$$= \frac{15V}{375\Omega}$$

$$= 0.040A$$

$$= 40\ \mathrm{mA}$$

# Current Divider

## Response 06(c).

Using the Current Divider formulation, then the current in the respective parallel branches are

$$I_1 = \left( \frac{R_2}{R_1 + R_2} \right) I_T \qquad I_2 = \left( \frac{R_1}{R_1 + R_2} \right) I_T$$

$$= \frac{600\Omega}{1600\Omega} \times 40mA \qquad = \frac{1000\Omega}{1600\Omega} \times 40mA$$

$$= 15\text{ mA} \qquad = 25\text{ mA}$$

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# Current Divider

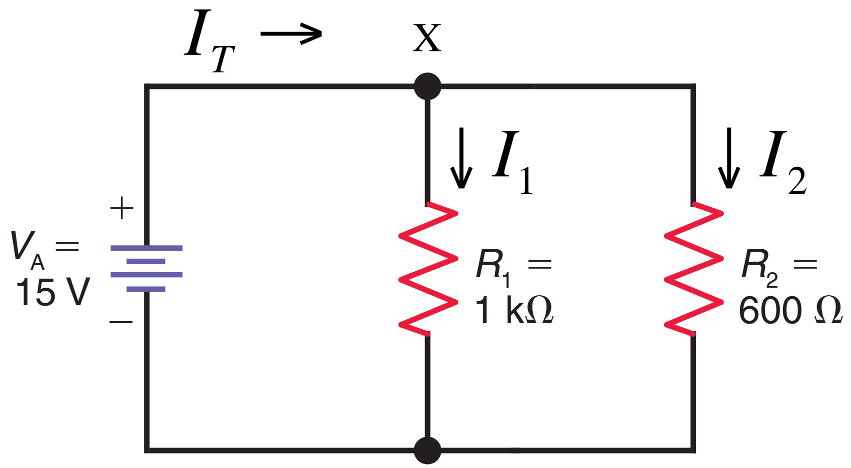
## Response 06(c). (cont.d)

Now, the algebraic sum of the currents at node X in the circuit is

$$\sum_{\text{At Node X}} I = I_1 + I_2 - I_T$$

$$= (15 + 25 - 40)\,mA$$

$$= 0$$



Therefore, KCL is verified.

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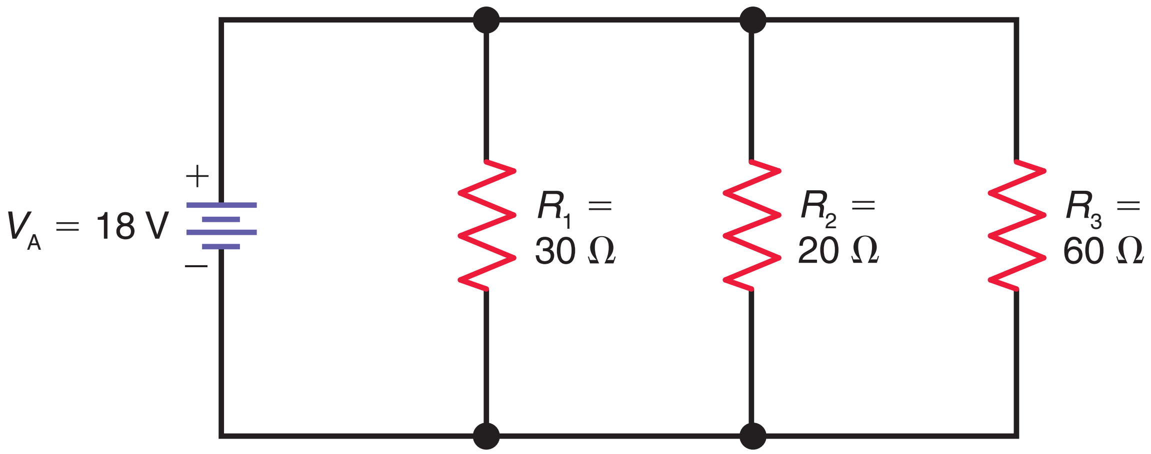
# Current Divider

## Example 07.

(a) Find $R_T$ , the total resistance of the network shown.

(b) Compute the total current from the 18V source.

(c) Compute the current through each each resistor.



# Current Divider

Parallel:

$$R_P = \left( \sum_{i=1}^{n} \frac{1}{R_i} \right)^{-1}$$

## Response 07(a).

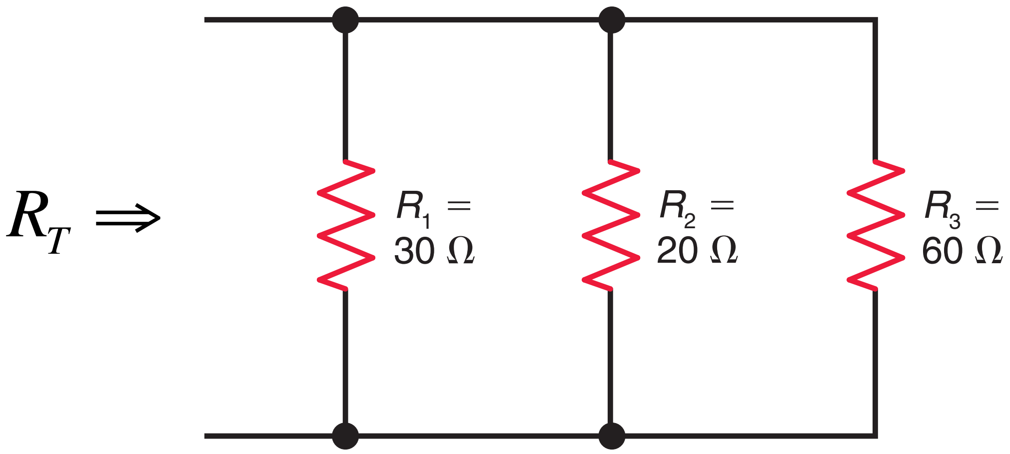
The total resistance of the parallel network is

$$R_T = \left( \sum_{i=1}^{3} \frac{1}{R_i} \right)^{-1}$$

$$= \left( \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right)^{-1}$$

$$= \left( \frac{1}{30} + \frac{1}{20} + \frac{1}{60} \right)^{-1} \Omega$$

$$= 10\ \Omega$$



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# Current Divider

Parallel:

$$R_P = \left( \sum_{i=1}^{n} \frac{1}{R_i} \right)^{-1}$$

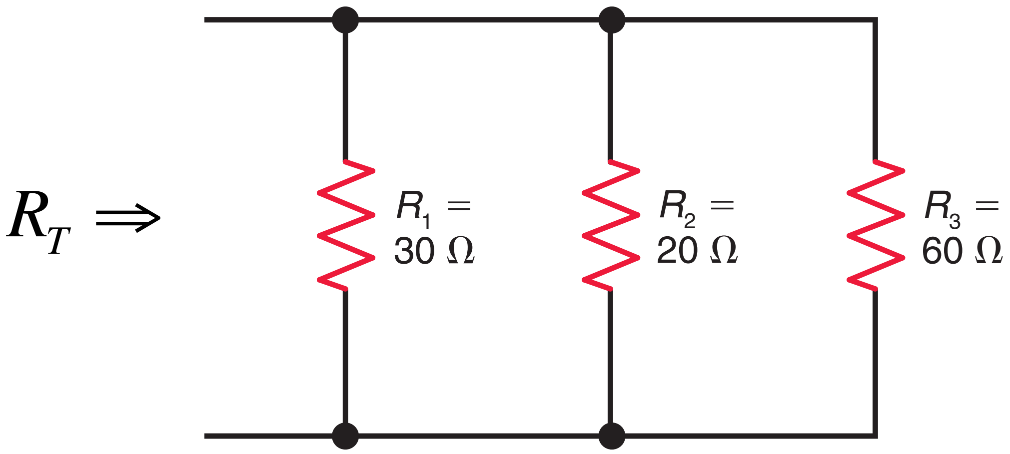
## Response 07(b).

The total current delivered by the source is

$$I_T = \frac{V_A}{R_T}$$

$$= \frac{18V}{10\Omega}$$

$$= 1.8\ \text{A}$$



# Current Divider

## Response 07(c).

Since all 3 resistors are in parallel with the source, then:

$$I_1 = \frac{V_A}{R_1} = \frac{18\ V}{30\ \Omega} = 0.6\ A$$

$$I_2 = \frac{V_A}{R_2} = \frac{18\ V}{20\ \Omega} = 0.9\ A$$

$$I_3 = \frac{V_A}{R_3} = \frac{18\ V}{60\ \Omega} = 0.3\ A$$

---

Reader: Using these results, please verify KCL:

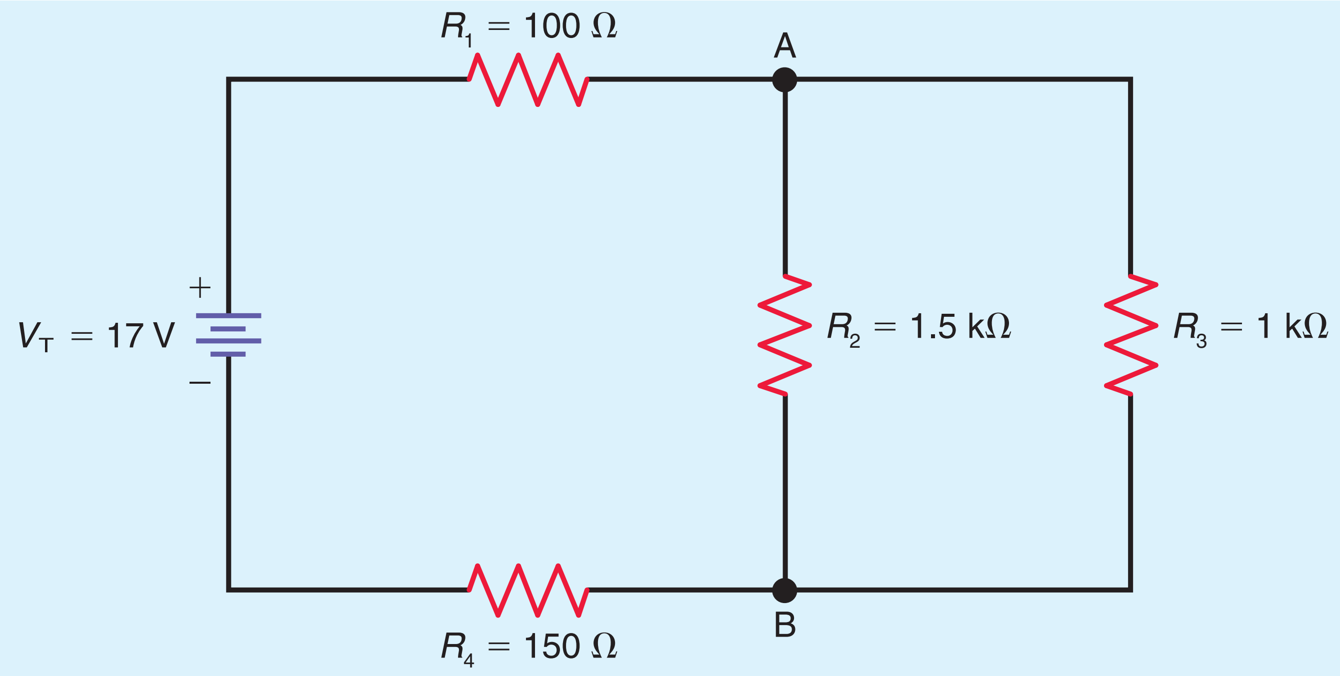
$$\sum_{at\ node} I = 0\,A$$

# Network Analysis

## Example 08.

(a) Find $R_{eq}$, the total resistance of the network shown.

(b) Compute $V_{AB}$ and the power loss in $R_3$.



# Network Analysis

## Response 08(a).

Using the formulations for resistors in series and parallel,

$$R_{eq} = R_1 + \frac{R_2 R_3}{R_2 + R_3} + R_4$$

$$= 100\Omega + \left(\frac{1.5 \times 1}{1.5 + 1}\right) k\Omega + 150\Omega$$

$$= 250\Omega + 600\Omega$$

$$= 850\ \Omega$$

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NB: (a) For 2 resistors in parallel, "product over sum" is quickly used.

(b) Please pay attention the mixed units and the final conversions!

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# Network Analysis

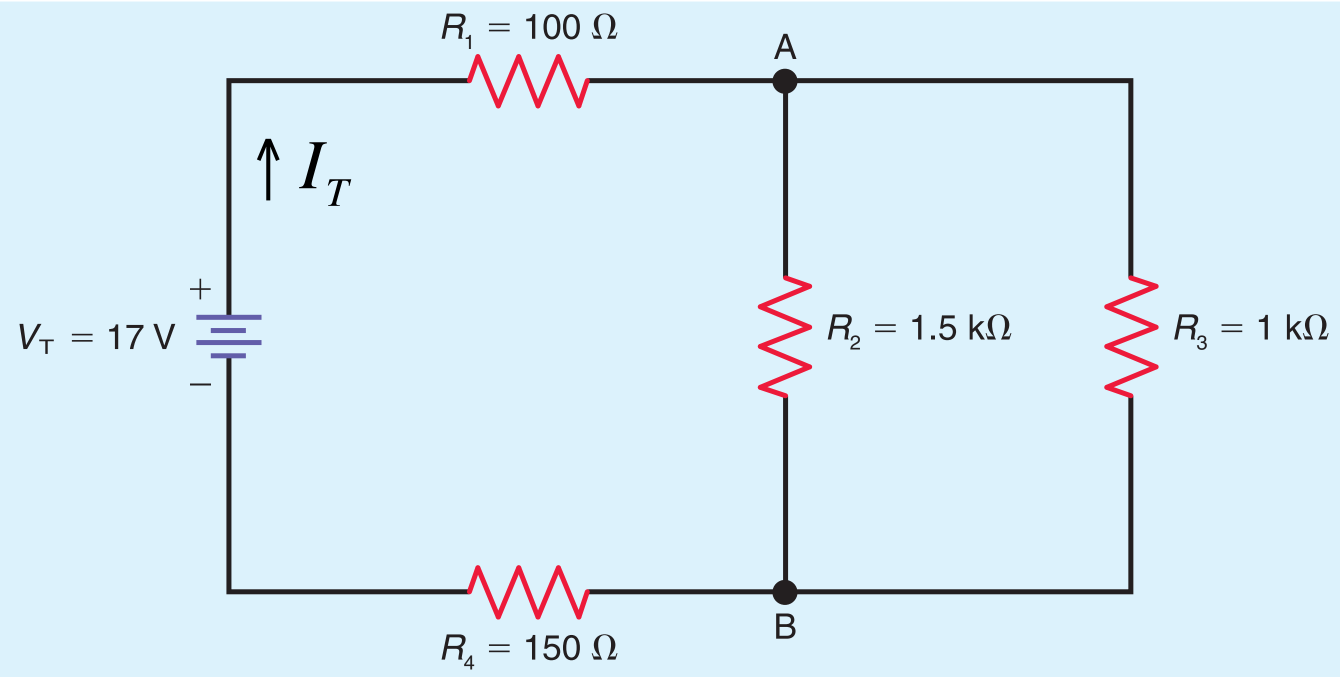
## Response 08(b).

First, find the total current delivered by the source is

$$I_T = \frac{V_T}{R_{eq}}$$

$$= \frac{17V}{850\Omega}$$

$$= 20\ \text{mA}$$



# Network Analysis

## **Response 08(b).** (cont.d)

Now, the total voltage drop across **R₁** & **R₄**, which are in *series*, is

$$\begin{aligned}\Delta V &= I_T\left(R_1 + R_4\right)\\ &= 20mA \times \left(100 + 150\right)\Omega\\ &= 5\text{V}\end{aligned}$$

So, the required p.d. across nodes A and B is

$$\begin{aligned}V_{AB} &= V_T - \Delta V\\ &= 17V - 5V\\ &= 12\ \text{V}\end{aligned}$$

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# Network Analysis

## **Response 08(b).** (cont.d)

And, finally, *power* loss in **R₃** is

$$P_3 = \frac{V_{AB}^2}{R_3} \qquad \text{(Joule's Law)}$$

$$= \frac{(12V)^2}{k\Omega}$$

$$= 144 \times 10^{-3} W$$

$$= 144 \text{ mW}$$

# R2S Legacy Tutorial 03

## Electric Circuit Analysis

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# Anatomy of the Cell

## Question 01.

The e.m.f. of a cell is **10.7 V**. When it is connected to a **12.0-Ω** load resistor, the terminal p.d. drooped by **10.0%**. Calculate

(a) the terminal p.d. of the cell, $V$.

(b) the load current $I$.

(c) the internal resistance of the cell, $r$.

(d) the rate of energy loss in the cell (i.e., power loss, $P_{loss}$ )

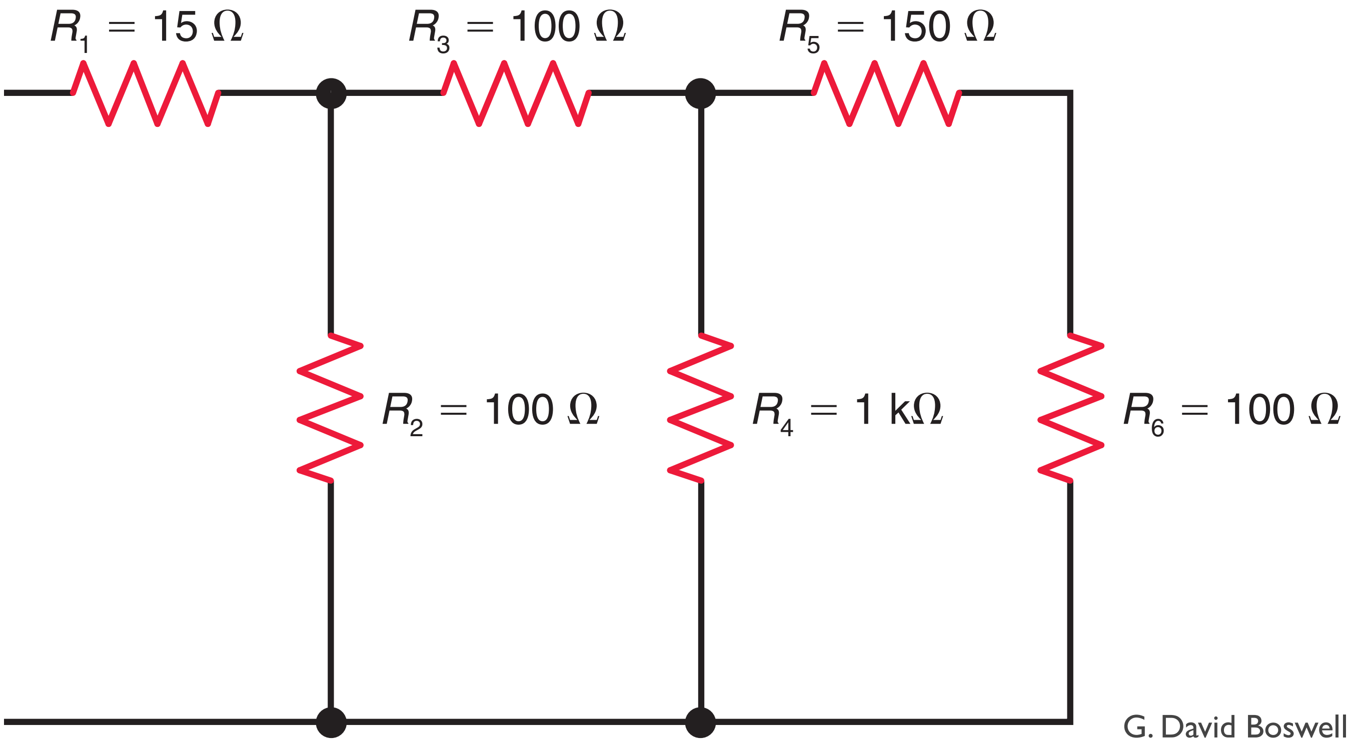
(e) the *fractional efficiency* of the cell.

# Network Reduction

## Question 02.

(a) Find $R_{eq}$, the equivalent resistance of the network shown.

(b) Repeat (a) when $R_3 = 0$ and $R_4 \to \infty$ simultaneously.

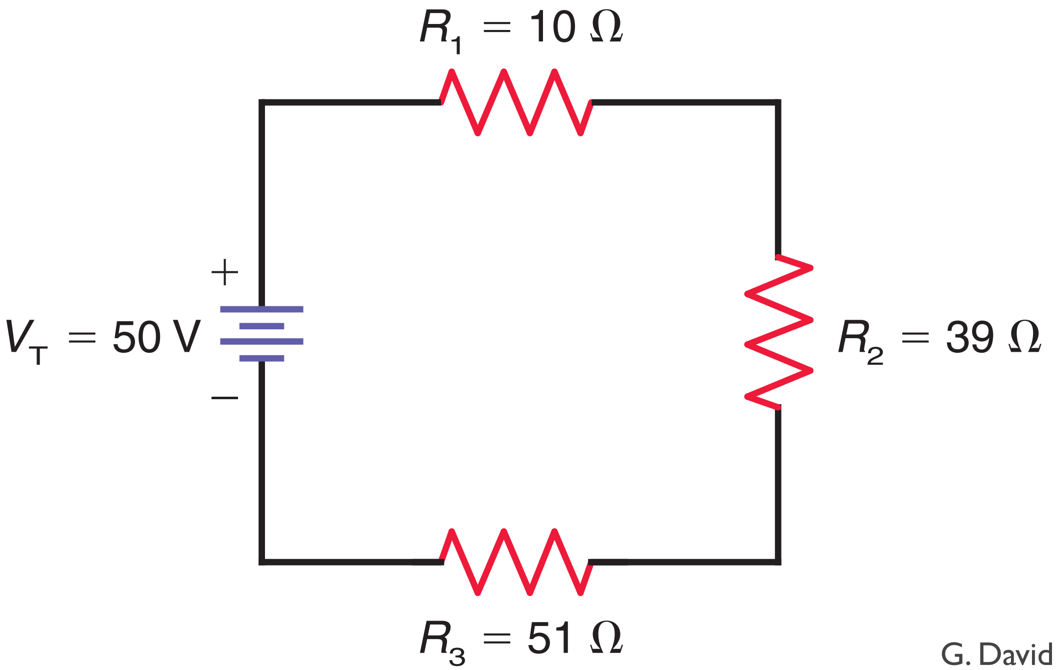


# Potential Divider

## Question 03.

(a) Find $R_T$, the total resistance of the network shown.

(b) Compute, $\mathbf{V_1}$, $\mathbf{V_2}$ and $\mathbf{V_3}$, the respective voltage drops across each resistor.



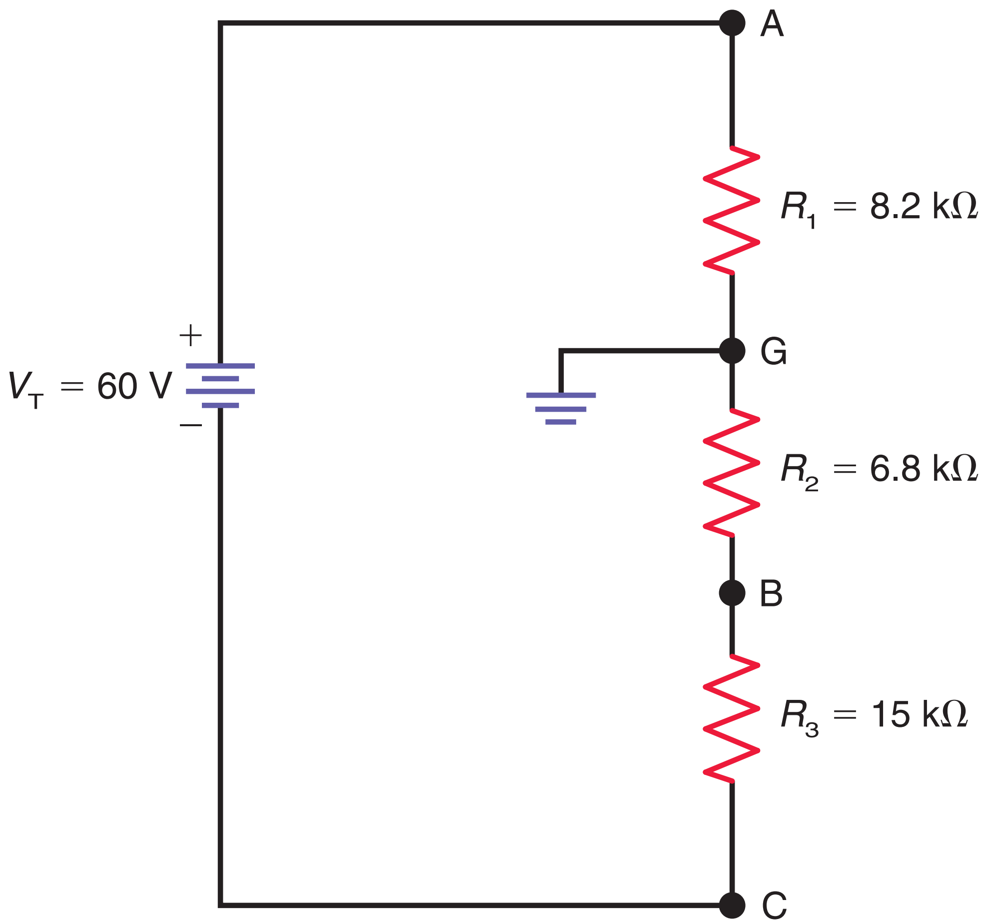
# Potential Divider

## Question 04.

(a) Find $R_S$, the total resistance of the network shown.

(b) Compute the p.d. across each resistor.

(c) Hence, calculate the **V$_C$** and **V$_B$**, the potentials at nodes **C** and **B**, respectively.

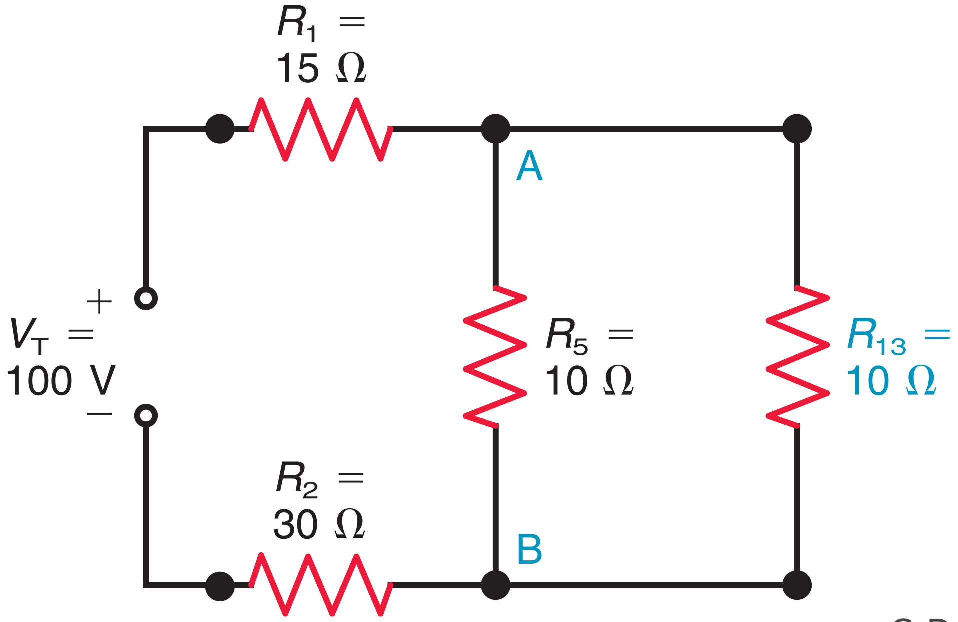


# Current Divider

## Question 05.

(a) Find $R_T$, the total resistance of the network shown.

(b) Hence, compute the total current supplied and also the current through each each of the 4 resistors.



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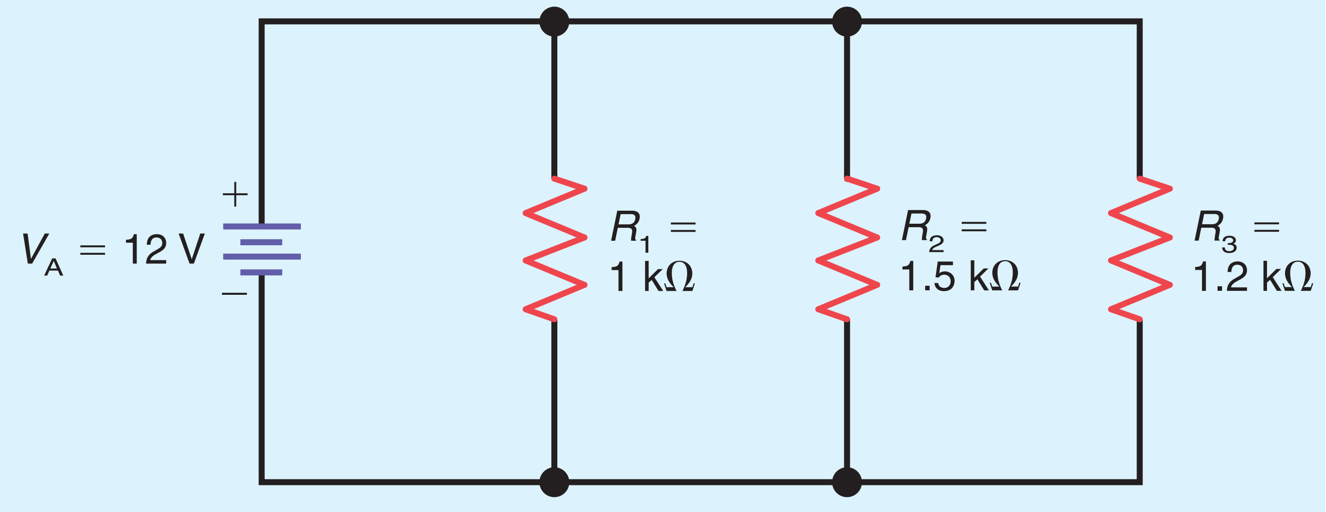
# Current Divider

## Question 06.

(a) Find $R_P$, the total resistance of the network shown.

(b) Compute the current supplied by the source voltage.

(c) Compute the current through each resistor and verify the Kirchhoff's Current Law (KCL) holds true.



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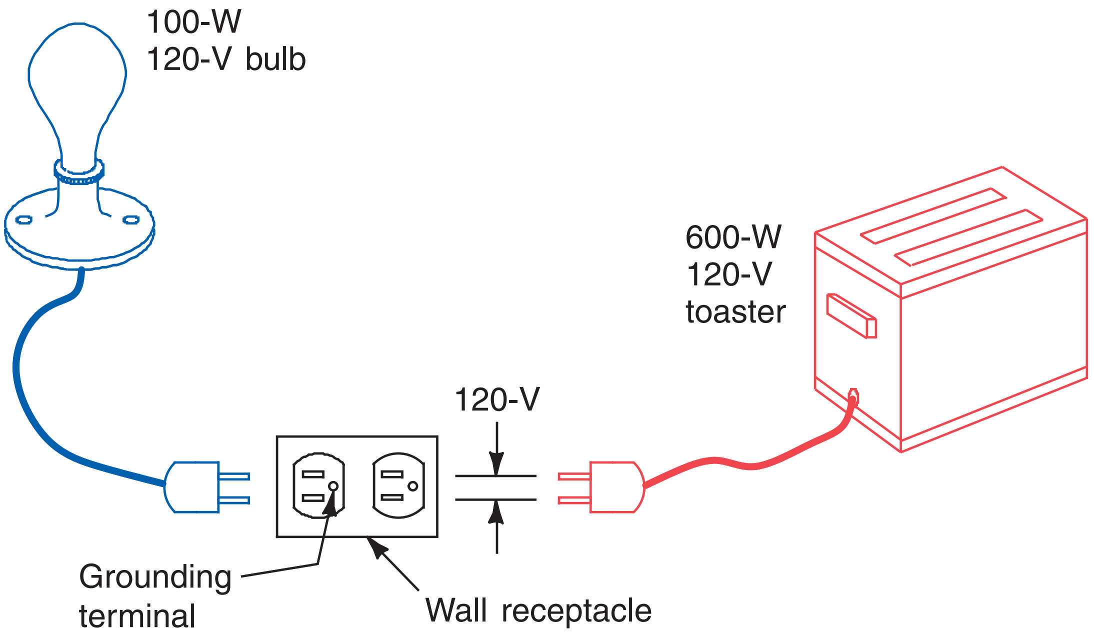
# Network Analysis

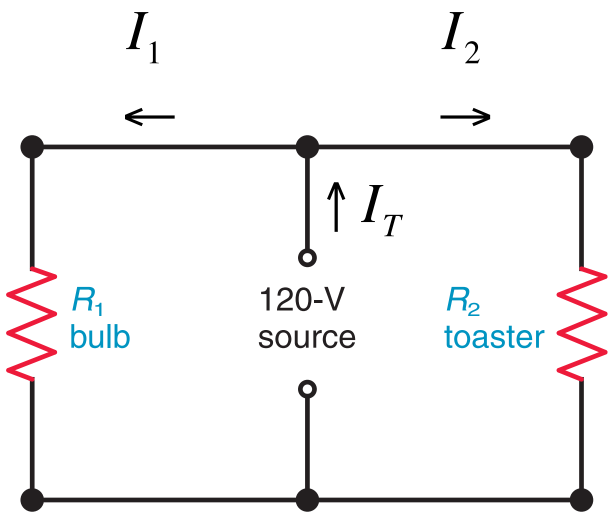
## Question 07.

(a) Find resistances $\mathbf{R_1}$ and $\mathbf{R_2}$ using ~~Ohm's and~~ Joule's Law.

(b) Use "Current Division" and Ohm's Law to find $\mathbf{I_1}$ and $\mathbf{I_2}$.



(*a*)



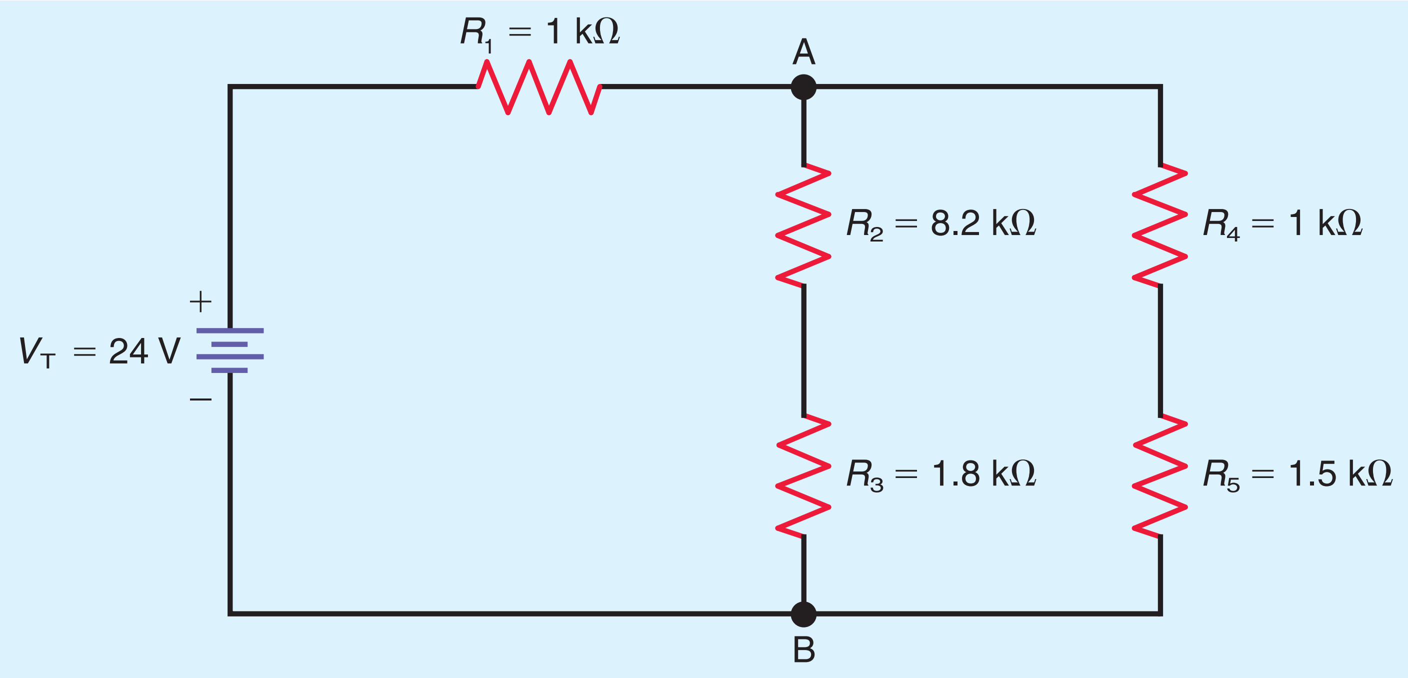
(*b*)

# Network Analysis

## Question 08.

(a) Find $R_{eq}$ , the total resistance of the network shown.

(b) Compute $V_{AB}$ and the *power* losses in $R_1$ and $R_5$ .



# R2S Legacy Tutorial 04

## Mesh and Nodal Analyses

10.12.21 … High Noon*

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# Electric Circuit Analysis

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