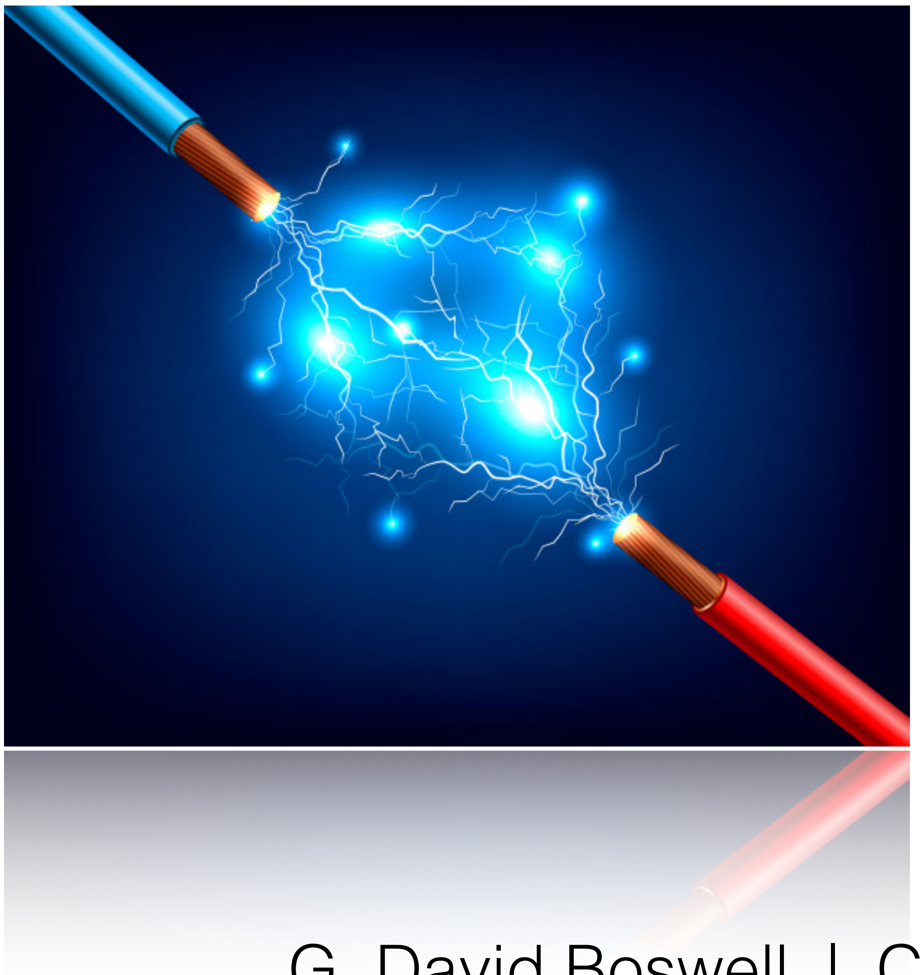
The Utility of Charge, q



TUTORIAL CONTENT

1. The Elementary Charge
2. Electric Current
3. Analysis of simple charge, q(t) & current, i(t) graphs
4. Work, Energy & Power

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The 21st Century

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Electric Charge

Example 01.

The total charge metal dome of a Van de Graaf's was *estimated* to be **−20.01 Coulombs.**

Estimate the total number electrons, **N**, that were added to the atoms close to the surface of the metal dome.

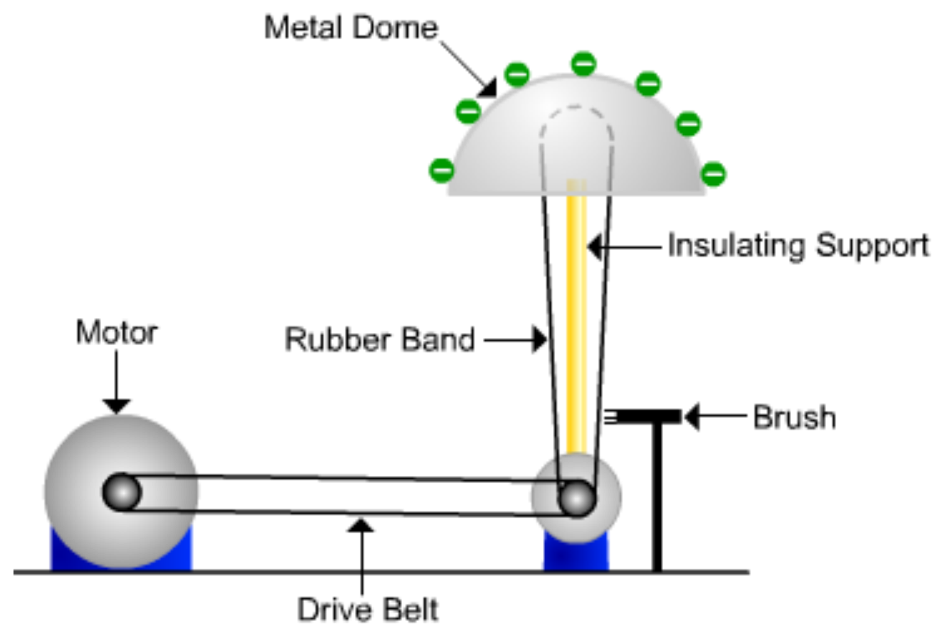

Diagram of charge-producing generator

Electric Charge

Response 01.

The total charge is $Q = Ne$. Hence, the required number of electrons is

$$N = \frac{Q}{e}$$

$$= \frac{-20.01}{-1.60 \times 10^{-19}} \frac{C}{C \text{ per } electron}$$

$$= 1.25 \times 10^{20} \text{ electrons}$$

(Please note the units and the appropriate number of sig. figs.)

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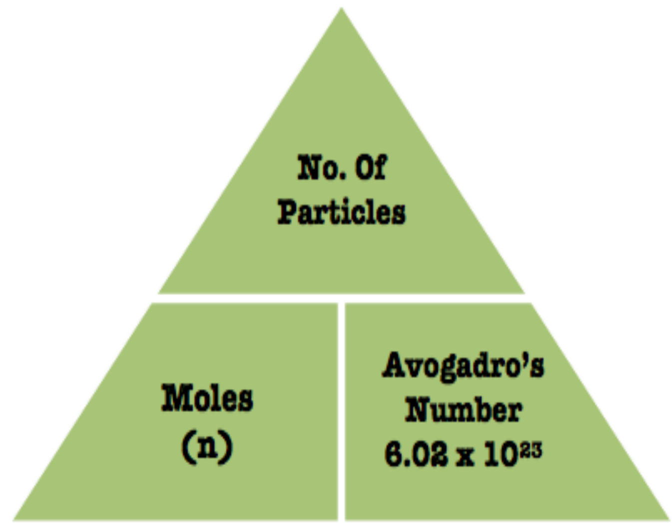
Electric Charge

Example 02.

How many moles of electrons constitutes a total charge of magnitude **Q = 0.55 C?**



The 'mole of triangle'

Data:

$$\text{Avogadro's number,}\quad N_A = 6.023 \times 10^{23}\ \text{particles} \cdot \text{mol}^{-1}$$

$$\text{Electronic charge,}\quad e = -1.60 \times 10^{-19}\, C$$

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Electric Charge

Response 02.

Recall the definition of the mole: $n = \dfrac{N}{N_A}$.

Then, using $Q = Ne$ and working with the magnitudes only, we get

$$n = \frac{N}{N_A}$$

$$= \frac{Q}{eN_A}$$

$$= \frac{0.55C}{\left(1.60\times10^{-19}C\right)\times\left(6.023\times10^{23}\,\text{mol}^{-1}\right)}$$

$$\approx 5.7\times10^{-6}\ \text{mol}$$

Reader:
Why is the final solution to 2 sig figs?

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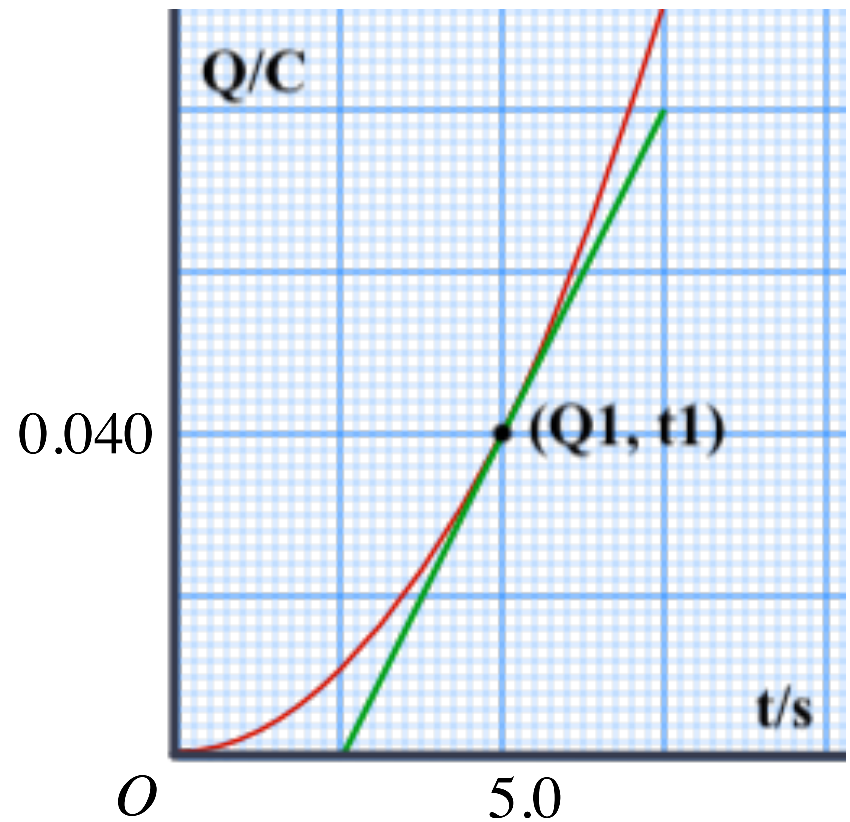
Electric Charge

Example 03.

Consider the graph of the time variations of charges flowing in a conductor shown.

Calculate the instantaneous value of the electric current at the time **t = 5.0 ms.**



Electric Charge

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Response 03.

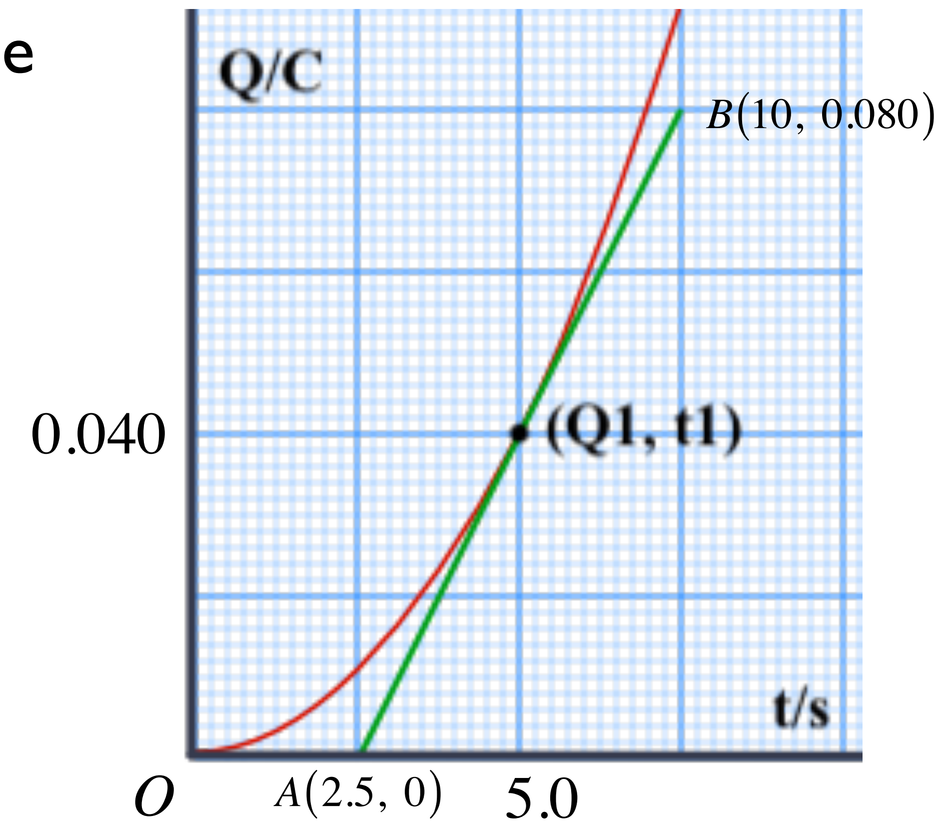
The instantaneous current is the slope of the **Q(t)** graph at time **t = 5.0 s.**

$$I = \frac{dQ}{dt} \quad \text{at } t = 5.0s$$

$$= \frac{Q_2 - Q_1}{t_2 - t_1} \quad \text{(using tangent AB)}$$

$$= \frac{(0.080 - 0)}{(10 - 2.5)} \frac{C}{s}$$

$$= 11 \text{ m}A$$



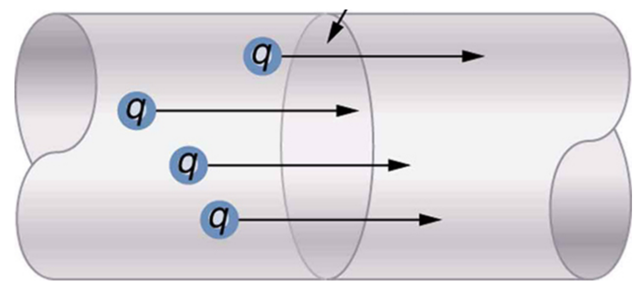
Electric Charge

Example 04.

Consider the passage of electric charges as they flow through a conducting medium.

- At $t = 0$, the total charge was **2.50 mC**.
- At $t = 12.5\ \mu s$, the total charge was **13.7 mC**.

Calculate the average current that was flowing through the medium during this time interval.



Electric charges in motion through a medium

Electric Charge

Response 04.

The average current that flowed is the gradient given by:

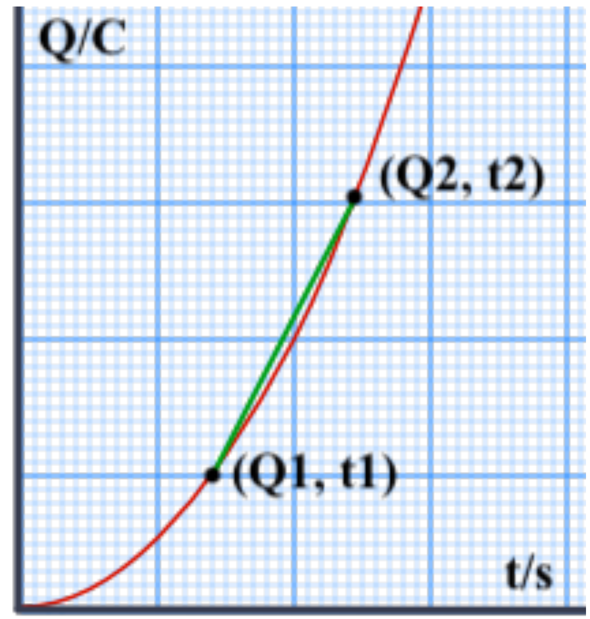
$$I = \frac{\Delta Q}{\Delta t}$$

$$= \frac{Q_2 - Q_1}{t_2 - t_1} \quad \leftarrow \text{optional step}$$

$$= \frac{(13.7 - 2.50)}{(12.5 - 0)} \frac{mC}{\mu s}$$

$$= 0.890 \times 10^3 \ C \cdot s^{-1} \quad \leftarrow \text{optional step}$$

$$= 89.0 \ A$$



Electric Charge

Example 05.

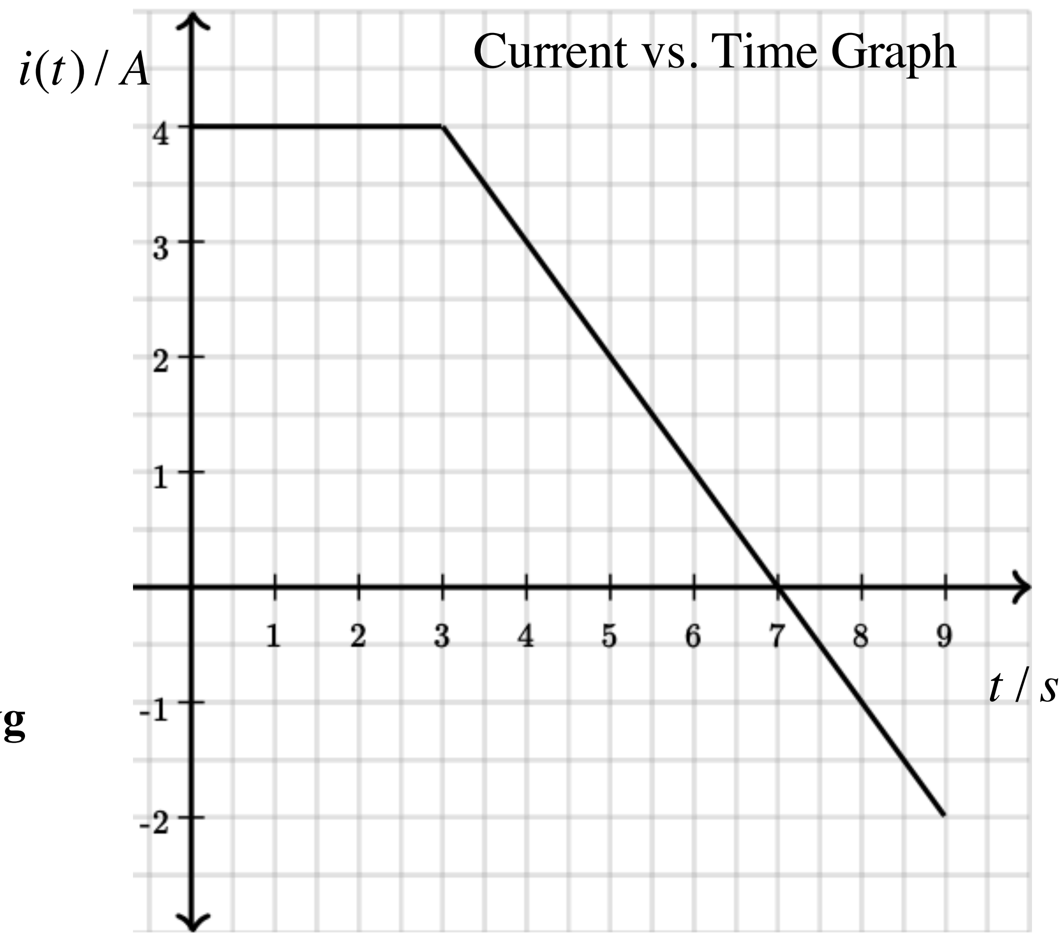
Consider a piecewise linear *Current versus Time* graph shown.

Calculate the following:

(a) The total charge, $\mathbf{Q_T}$

(b) The average current, $\mathbf{I_{avg}}$

(c) The r.m.s. value of the current, $\mathbf{I_{rms}}$ (A bonus question for the reader!)



The i(t) graph showing a *non-uniform* current

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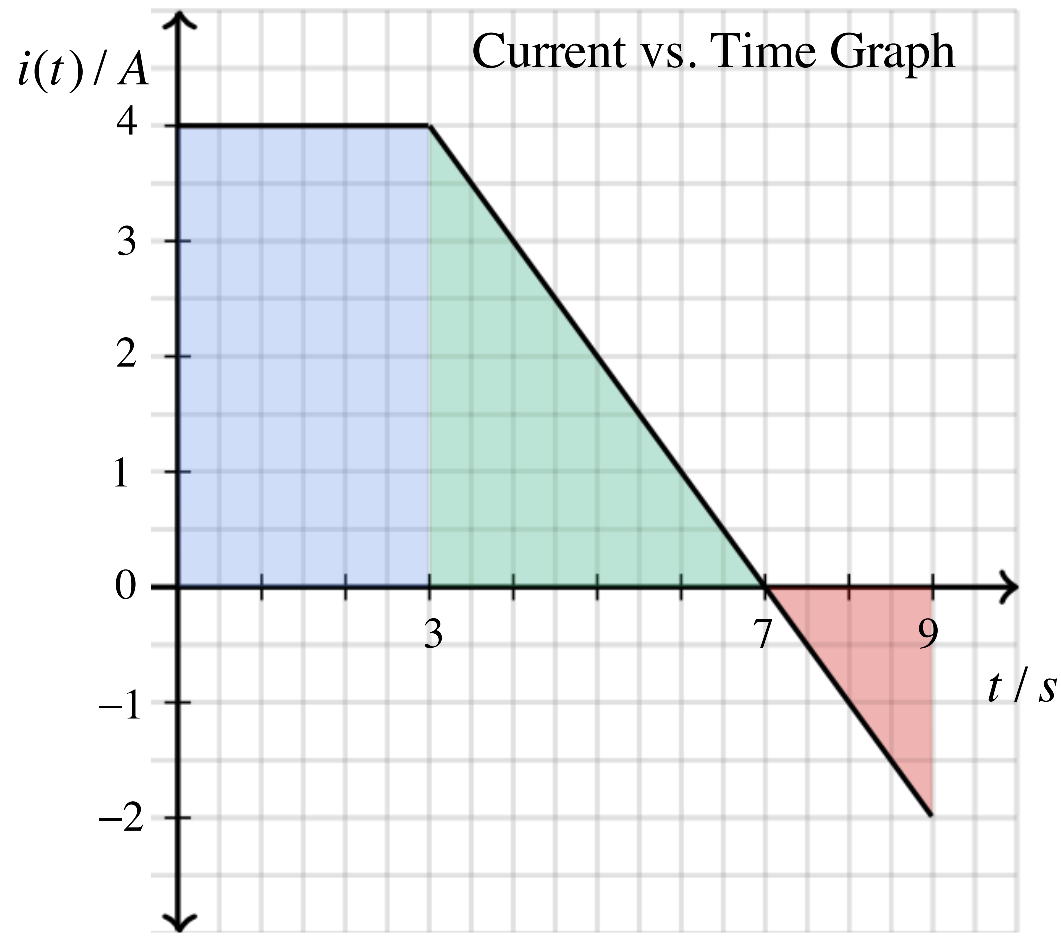
Electric Charge

Response 05.(a)

The total charge is the "trapped *area of the current vs time graph*." Mathematically, using calculus, it is

$$Q_T = \int_0^9 i(t)\,dt$$

Geometrically, it is the total area of the colored regions!



The i(t) graph showing a *non-uniform* current

Electric Charge

Response 05.(a) (cont'd)

So, using the *geometry of the graph*, the total charge is:

$$Q_T = 4(3-0) + \frac{1}{2}(4)(7-3) + \frac{1}{2}(-2)(9-7) \quad \mathrm{A \cdot s}$$
$$= 12 + 8 - 2 \quad C$$
$$= +18 \ C$$

Please note that this sum is "algebraic." That is, during any time interval when the current is positive or negative, the polarity of the charge is positive or negative, respectively. Also, $1 \mathrm{A \cdot s} = 1 \ \mathrm{C}$

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Electric Charge

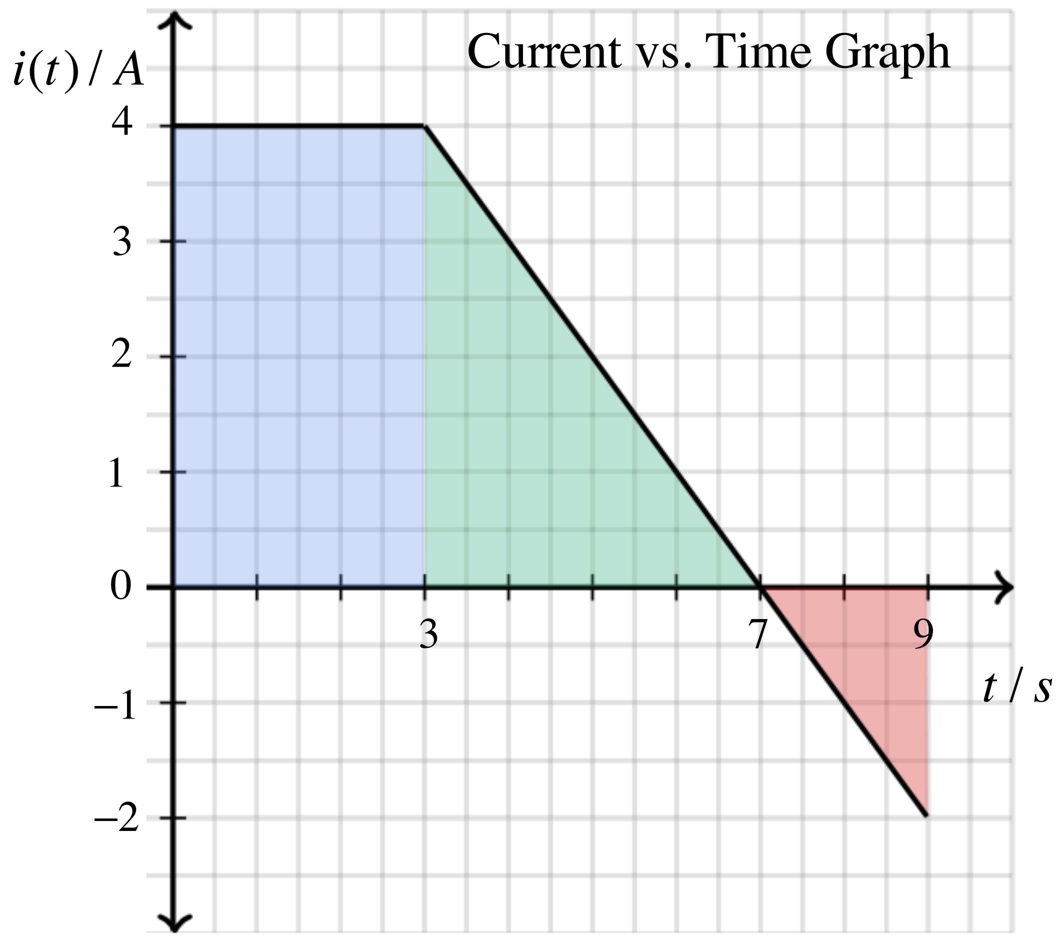
Response 05.(b)

There are 3 ways to compute the average current.

Approach I ~ *Using Calculus*

$$I_{avg} = \frac{1}{t_2 - t_1}\int_{t_1}^{t_2} i(t)\,dt$$

But … I will leave this method for Zoom Discussions!



The i(t) graph showing a *non-uniform* current

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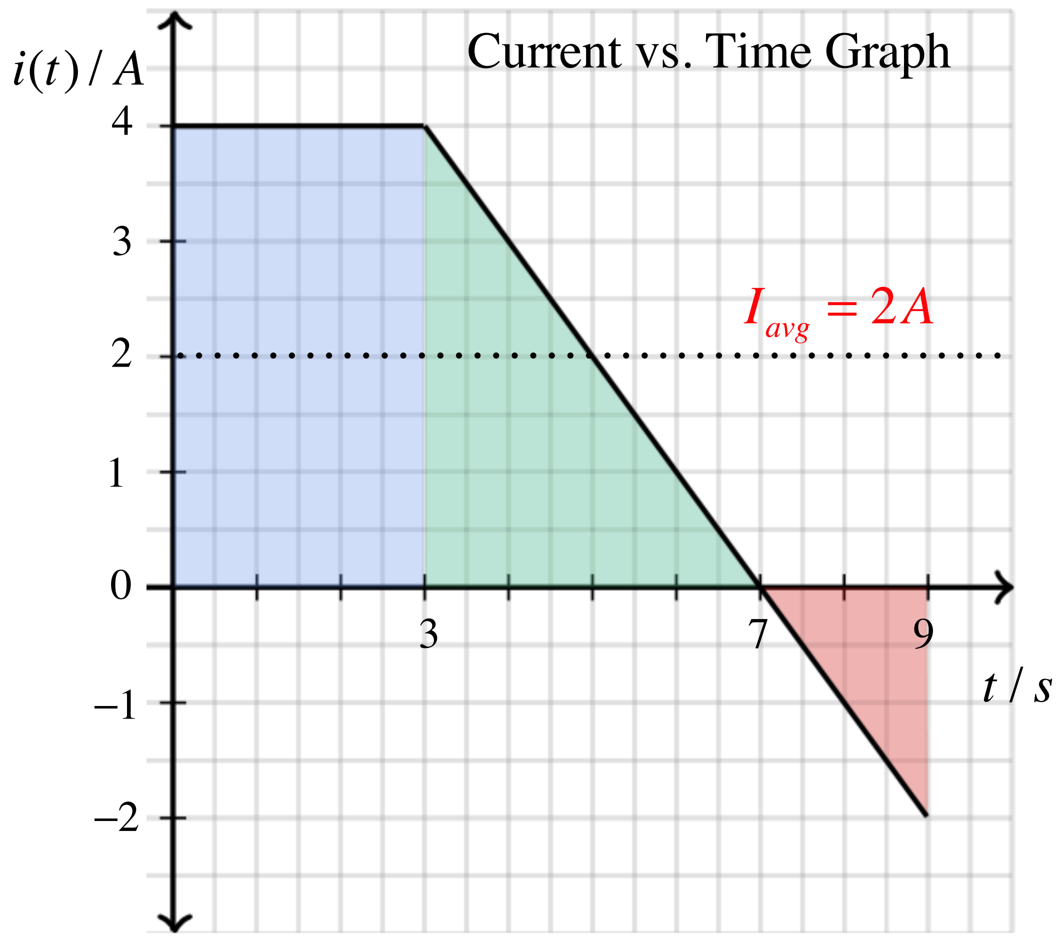
Electric Charge

Response 05.(b) (cont'd)

There are 3 ways to compute the average current.

Method II ~ *Using the Definition*

$$I_{avg} = \frac{\Delta Q}{\Delta t} = \frac{18C}{9s} = 2A$$



The i(t) graph showing a *non-uniform* current

Electric Charge

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Response 05.(b) (cont'd)

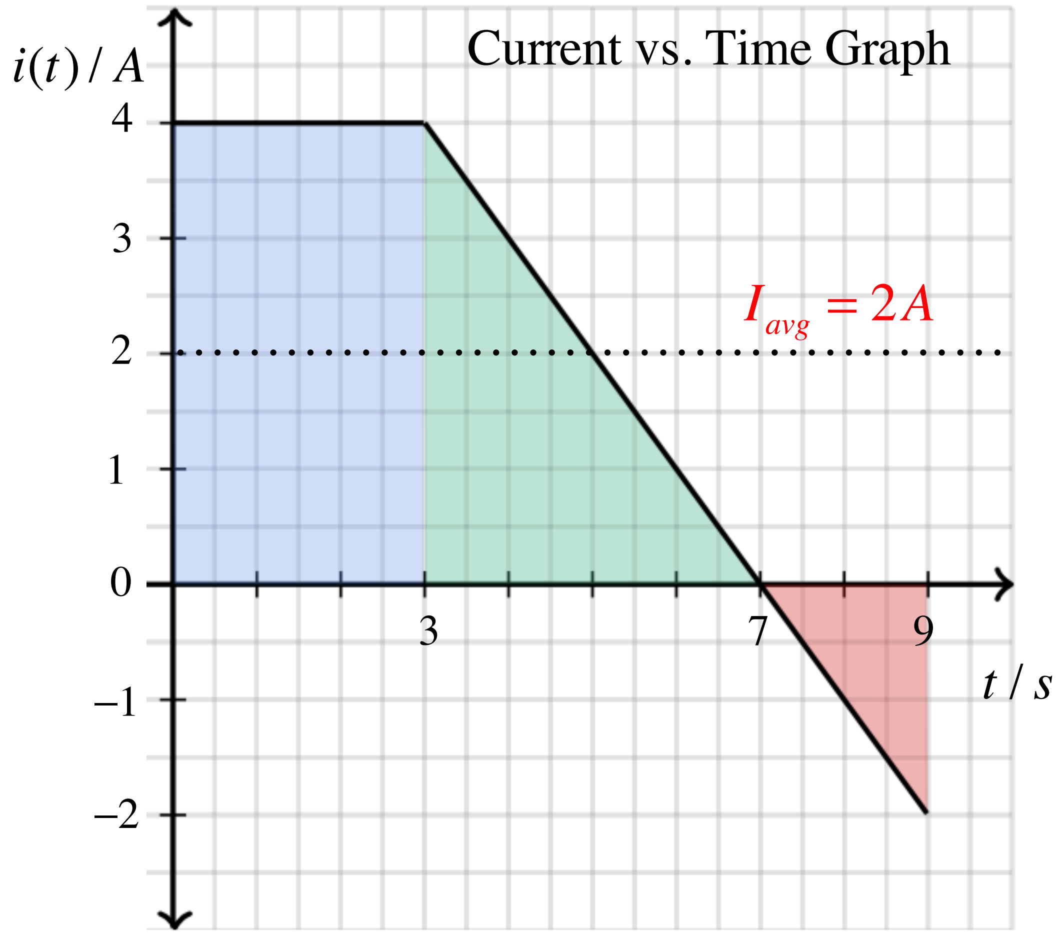
There are 3 ways to compute the average current.

Method III ~ *Using the Graph*

$$I_{avg} = \frac{1}{9}\left[4(3) + \frac{6}{2}(4 + (-2))\right] A$$

$$= \frac{1}{9}[12 + 6]\ A$$

$$= 2\ A$$



The i(t) graph showing a *non-uniform* current

Electric Charge

Example 06.

A standard mobile phone lithium Ion rechargeable "battery is rated at **3.7 V**".

(a) What is the meaning of the battery rating value of **3.7 V**?

(b) If your phone required **2.3 mW** of power for a ZOOM class for a duration of **10 mins**, how much charge was transferred?

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Electric Charge

Response 06(a). *This updated explanation is very important!*

Based on the definition of ***potential difference (p.d.)*** and the application of the ***Principle of Conservation of Energy***, the rating of **3.7 volts** for the lithium ion battery means that when it delivers **1 coulomb** of charge, **3.7 joules** of electrical energy will be converted to other form(s) of energy.

(Of course, if asked, the electromotive force (e.m.f.) of the battery would be greater than 3.7 V since energy loss as heat due to the *internal resistance* of the battery must be taken into account.)

Electric Charge

Response 06(b).

Recall, that on average, $\Delta V = \dfrac{\Delta W}{\Delta Q}$. So, when **V** is constant and using the relationship among energy, power and time, we get:

$$\begin{aligned}\Delta Q &= \frac{\Delta W}{V} \\ &= \frac{P\Delta t}{V} \\ &= \frac{2.3\ \text{mW} \times 10\ \ \text{min}}{3.7\ \ V} \times \frac{60\ \ \text{s}}{1\ \ \text{min}} \\ &= 373.0\ \ \text{mC} \\ &\approx 373\ \text{mC}\end{aligned}$$

Reader:
Why is the final solution to 2 sig figs?

Tutorial 01
Electric Charge

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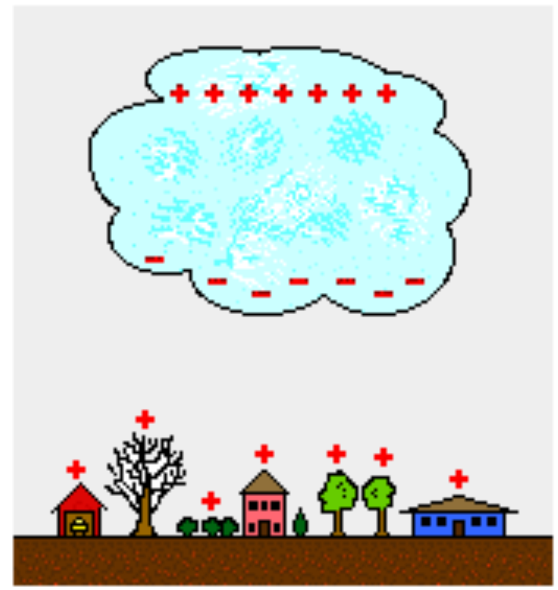
Electric Charge

Question 01.

Due to *frictional charging and induction,* one billion electrons were displaced in an initially uncharged small body of clouds poised for a lightning strike.

Estimate ΔQ, the total displaced charge of due to the movement of electrons. (Express your answer in the S.I. units of coulombs.)



Depiction of a polarized cloud

Electric Charge

Solution T.01

The total displaced charge due to the electrons in the *electrostatic phenomenon* (i.e., the onset of lightning) is:

$$\Delta Q = Ne$$

$$= (1 \text{ billion } electrons) \times \left(-1.60 \times 10^{-19} \ C \text{ per } electron\right)$$

$$= 10^{9} \times \left(-1.60 \times 10^{-19}\right) \ C$$

$$= -1.60 \times 10^{10} \ \text{C}$$

Electric Charge

Question 02.

How many moles of protons constitute a total charge of

(a) $Q_1 = +0.55$ C?

(b) $Q_2 = +100$ nC?

Data:

$$\text{Avogadro's number,} \quad N_A = 6.023 \times 10^{23} \text{ particles} \cdot \text{mol}^{-1}$$

$$\text{Proton charge,} \quad q = -e$$

$$= +1.60 \times 10^{-19} C$$

Electric Charge

Solution T.02(a)

Using the mole concept, $n_1 = \dfrac{N_1}{N_A}$. And, by using $Q_1 = N_1 q$, then the number of moles is:

$$
\begin{aligned}
n_1 &= \frac{N_1}{N_A} \\
&= \frac{Q_1}{qN_A} \\
&= \frac{+0.55C}{\left(+1.60\times10^{-19}C\right)\times\left(6.023\times10^{23}\,\text{mol}^{-1}\right)} \\
&\approx 5.7\times10^{-6}\ \text{mol}
\end{aligned}
$$

Electric Charge

Solution T.02(b)

Similarly, using the mole concept once more, then:

$$n_2 = \frac{N_2}{N_A} \qquad \text{with } \; Q_2 = N_2 q$$

$$= \frac{Q_2}{qN_A}$$

$$= \frac{+100 \times 10^{-9}\ C}{\left(+1.60 \times 10^{-19} C\right) \times \left(6.023 \times 10^{23} \text{mol}^{-1}\right)}$$

$$\approx 1.04 \times 10^{-12}\ \text{mol}$$

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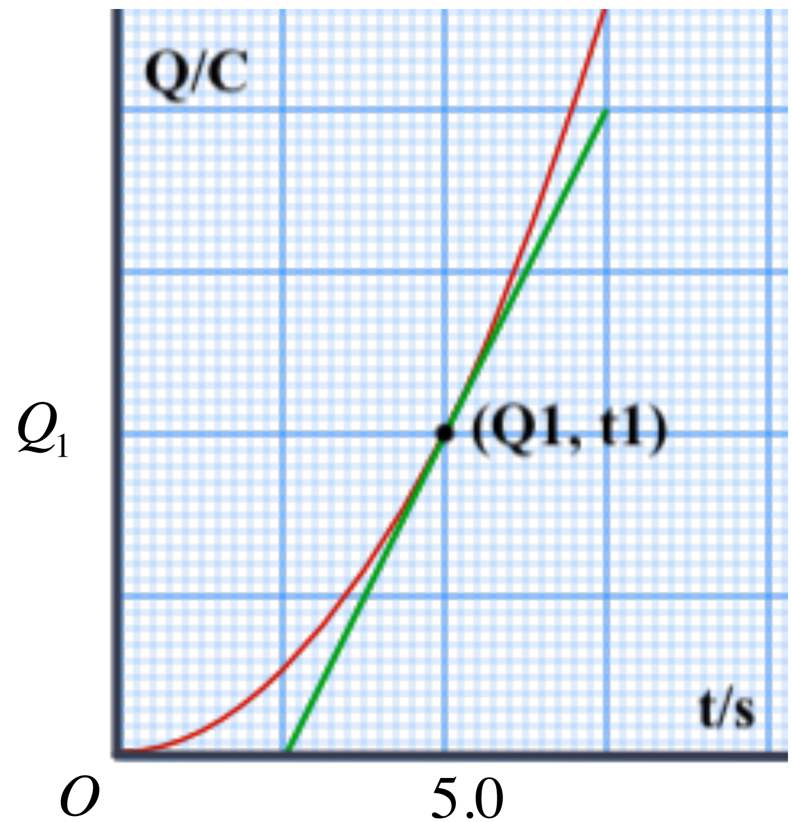
Electric Charge

Question 03.

Consider Q *vs.* t graph as shown.

The instantaneous value of the electric current at time, $t_1 = 5.0\text{s}$ is $i(\text{t}) = 14\ \text{A}$.

Calculate Q_1, the value of the charge at t_1 .



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Electric Charge

Solution T.03

The instantaneous current is the slope of the **Q(t)** graph at time **t = 5.0 s.**

So, using *Tangent AB*,

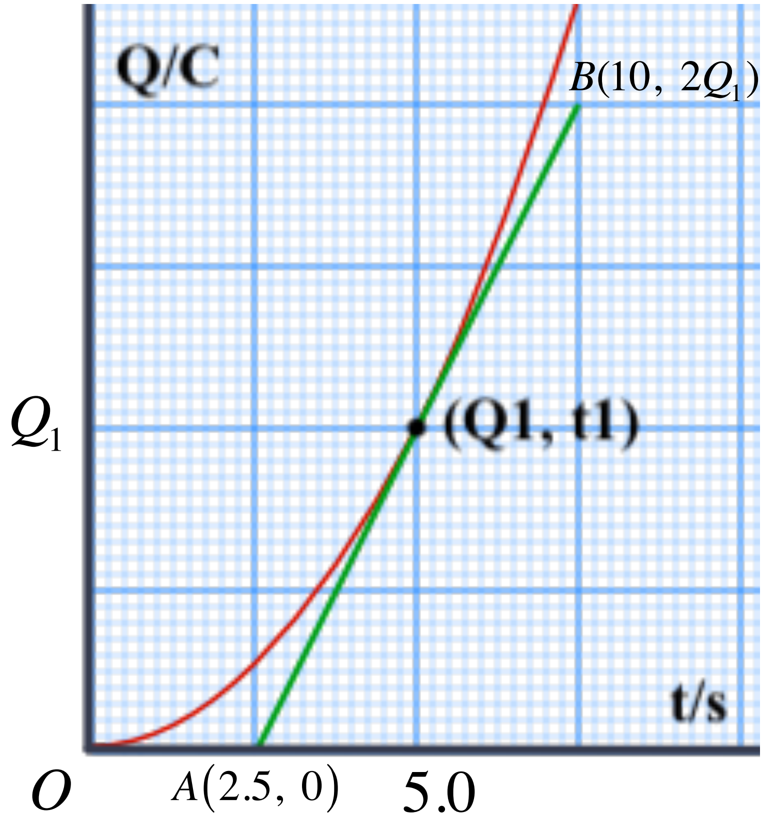
$$I = \frac{\Delta Q}{\Delta t}$$

Hence,

$$\Delta Q = I \Delta t$$

$$Q_1 - 0C = 14A \times (5.0 - 2.5)s$$

So, $Q_1 = 35\ C$



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Electric Charge

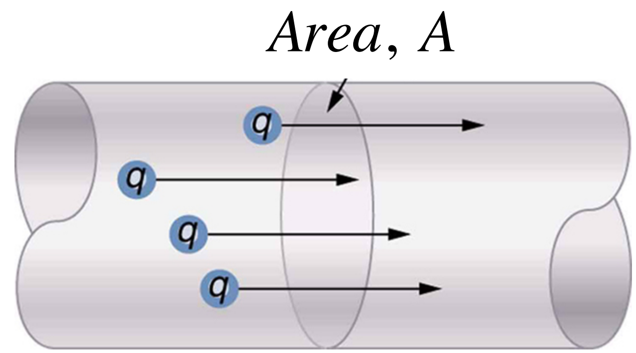
Question 04.

A conducting wire of radius 4.50 mm transmitted a uniform current of magnitude 10.0 A for 1 hour. Calculate

(a) the total charge, Q (in C) that flowed through the wire;

(b) the cross-sectional area of the circular wire, A (in m²);

(c) the current density, J (in A/m²).



Electric charges passing through a wire

Electric Charge

Solution T.04(a)

The total charge is $Q = It$

$$= 10.0A \times \left(1hr \times \frac{3600s}{1hr}\right) \quad \text{(units conversion)}$$

$$= 3.60 \times 10^{4}\ C$$

Solution T.04(b)

The required area is $A = \pi r^{2}$

$$= \pi \times \left(4.50 \times 10^{-3} m\right)^{2}$$

$$= 6.36 \times 10^{-5}\ m^{2}$$

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Electric Charge

Solution T.04(c)

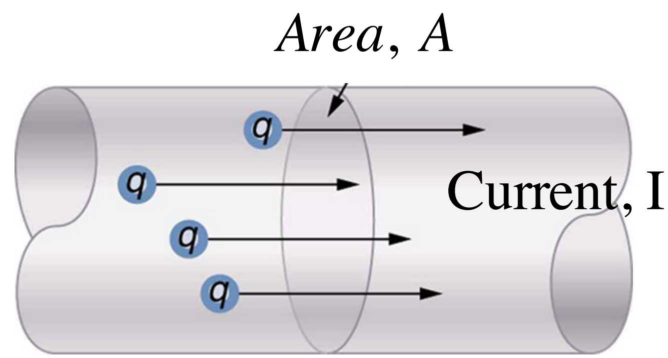
The ***current density*** of the wire is:

$$J = \frac{I}{A}$$

$$= \frac{10.0A}{6.36 \times 10^{4} m^{2}}$$

$$= 1.57 \times 10^{5} \ A \cdot m^{2}$$

(In this scenario, the area is orthogonal to the direction of the current).



Electric charges passing through a wire

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Electric Charge

Question 05.

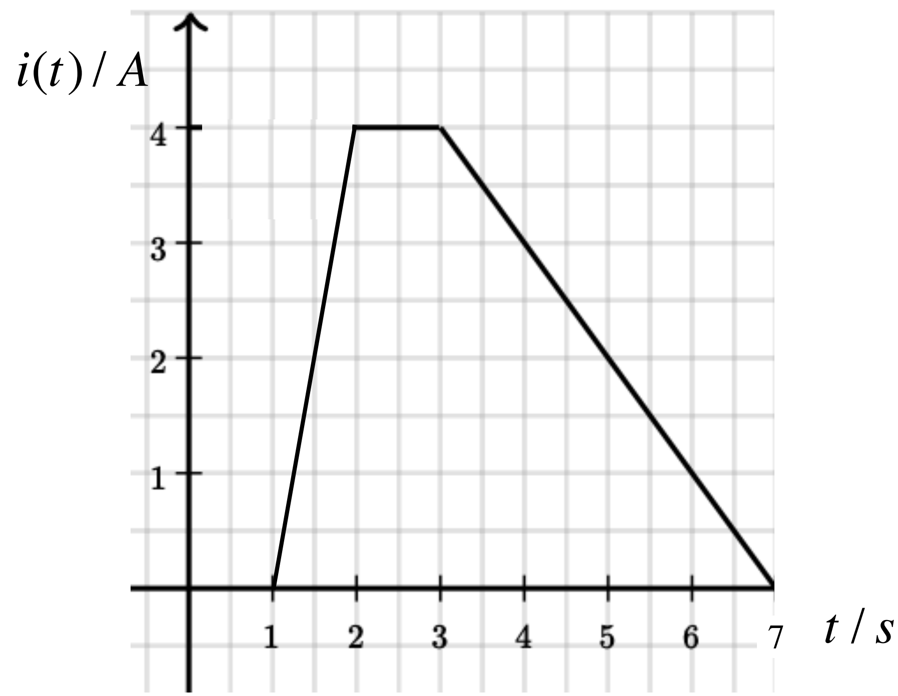
Consider a piecewise linear *Current versus Time* graph shown.

For the span of 0 to 7 seconds, compute the following:

(a) Total charge, $\mathbf{Q_T}$

(b) Average current, $\mathbf{I_{avg}}$

Current vs. Time Graph



Electric Charge

Solution T.05(a)

For 0 to 7s, the accumulation of charges (integration) is the trapped area* on the i(t) graph.

So,

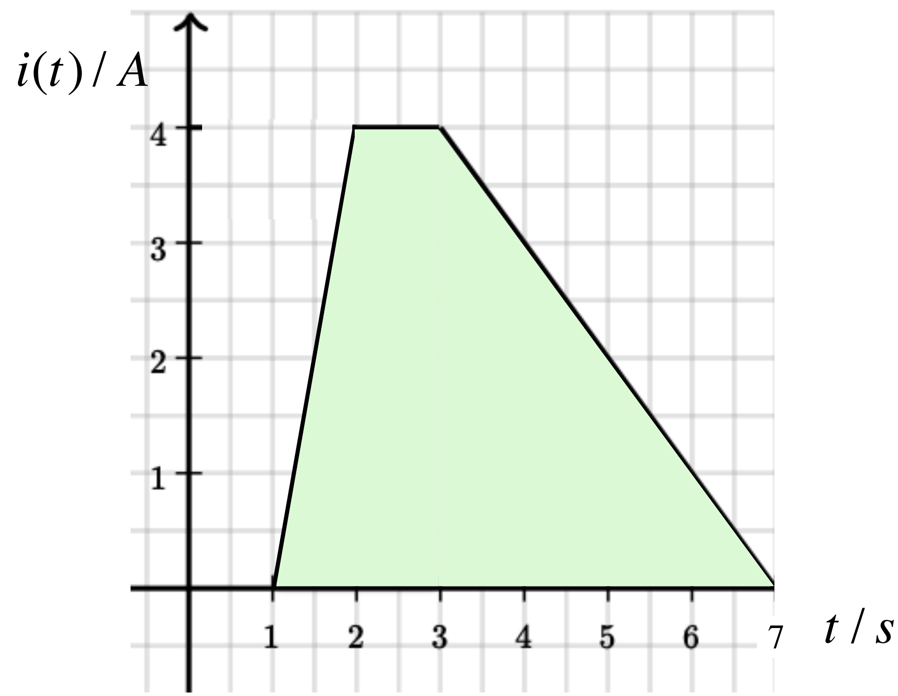
$$Q_T = \int_0^7 i(t)\,dt$$

$$= \frac{1}{2}(4A)\left[(3-2)+(7-1)\right]s$$

$$= 2(1+6) \quad A\cdot s$$

$$= +14\ C$$

Current vs. Time Graph



*The region of interest is a trapezium.

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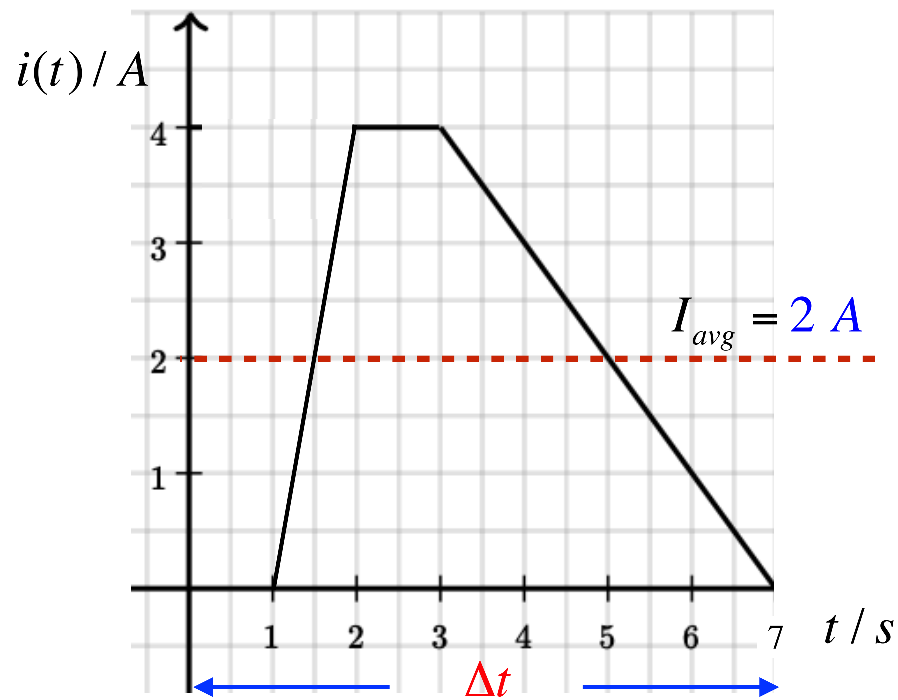
Electric Charge

Solution T.05(b)

Hence, the ***average current***, which is the *ratio of the total charge to the total time*, is computed as follows:

$$I_{avg} = \frac{Q_T}{\Delta t} = \frac{14C}{7s} = 2\ A$$

Current vs. Time Graph



Electric Charge

Question 06.

A battery is rated at "**3.7 V, 1100 mAh**".

When used for continuously for 2 hours without recharging, calculate

(a) the average current (I in mA) that it delivers.

(b) the total charge used (ΔQ in C).

(c) the total energy conversion (ΔW in mJ).

Electric Charge

Solution T.06(a)

The average current is
$$I_{avg} = \frac{Storage\ \text{Capacity}}{\Delta t}$$
$$= \frac{1100}{2}\frac{\text{mA}\cdot\cancel{\text{h}}}{\cancel{\text{h}}}$$
$$= 550\ \text{mA}$$

Solution T.06(b)

The total charge is
$$\Delta Q = I\Delta t$$
$$= 550\times10^{-3}A\times\left(2\cancel{hr}\times\frac{3600\,s}{1\ \cancel{hr}}\right)$$
$$= 3960\ C$$

Electric Charge

Solution T.06(c)

Since "voltage is work done per unit charge," then, mathematically,

$$\Delta V \triangleq \frac{\Delta W}{\Delta Q}$$

So, since V is constant,

$$\begin{aligned} \Delta W &= V\Delta Q \\ &= 3.7V \times 3960C \\ &= 1.46\times10^{4}\,J \times \frac{10^{3}\,mJ}{1J} \quad \text{(units conversion)} \\ &\approx 1.5\times10^{7}\,mJ \qquad \text{(2 sig. figs.)} \end{aligned}$$

Electric Charge

Thank You

For more information, please visit:
http://www.boszik.net/projects