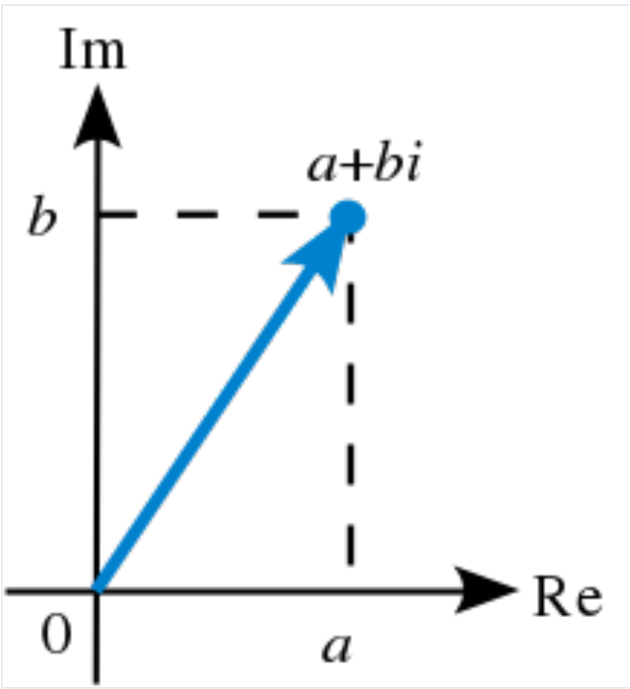
Complex Numbers



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The 21st Century

Definition of the Complex Number, z

It starts with an axiomatic development for the first '*imaginary number*' is:

$$\exists i: \quad i^2 = -1 \quad \Leftrightarrow \quad i = \sqrt{-1}$$

This permitted mathematicians and scientists to solve problems such as:

$$(a) \qquad x = \sqrt{-17}$$

$$(b) \qquad x^2 + 100 = 0$$

$$(c) \qquad x^2 + x = -1$$

Definition of the Complex Number, z

A Complex Number, **z**, is any number that can be represented on the Complex Plane.

It has a real part (Re(z)) and an imaginary part (Im(z)).

Thus, a complex number takes the form:

$$\text{Complex Number, } z = \text{Re}(z) + \left(\text{Im}(z) \times i\right)$$

$$\text{That is to say, } z = a + bi$$

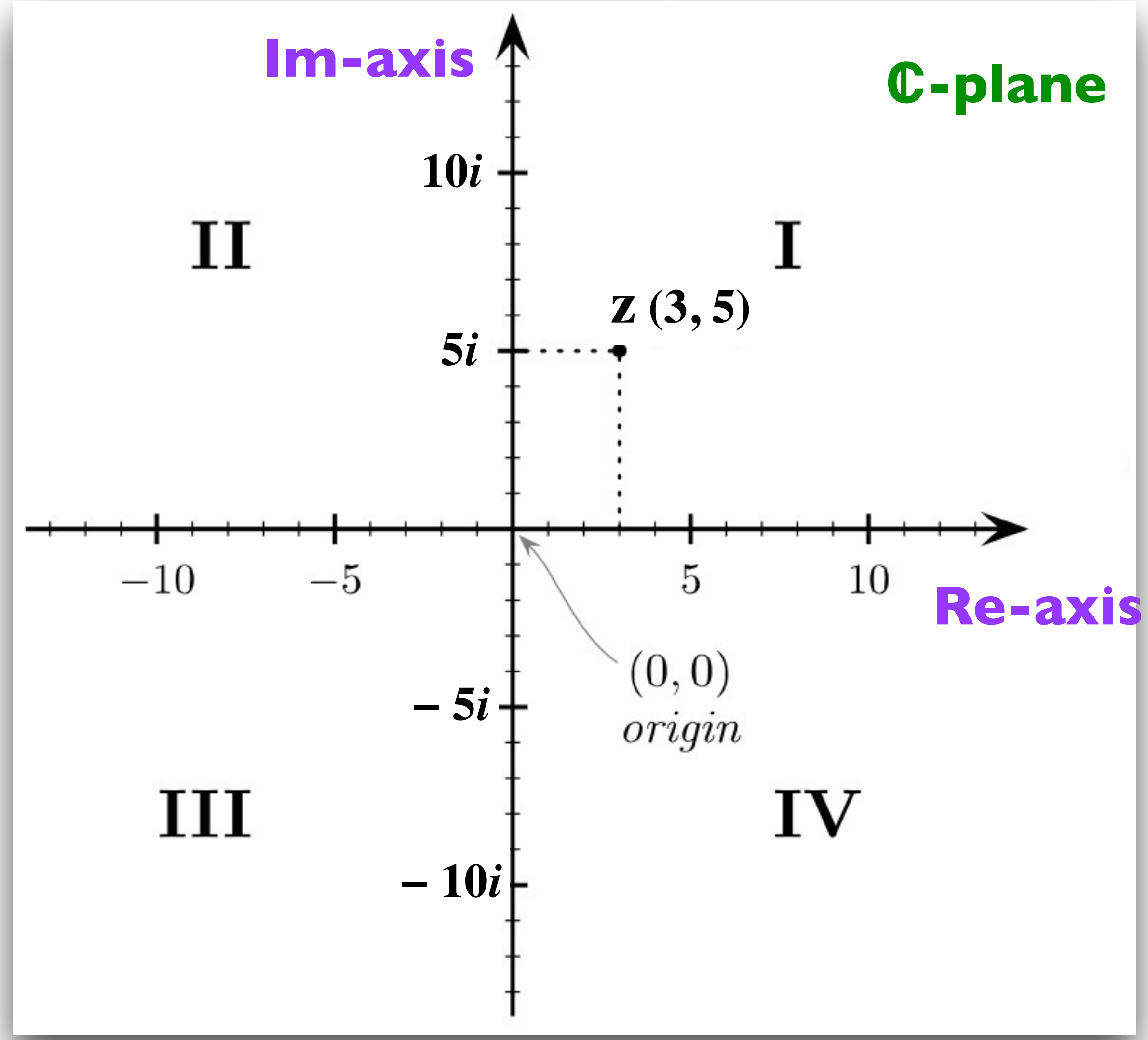
$$\text{where } a, b \in \mathbb{R}$$

The Argand Diagram (ℂ-plane)

A novel method first developed to *geometrically interpret complex numbers.*

For example, the Complex number shown is the *vector:*

$$z = 3 + 5i$$



Representations of ℂ-Numbers

Every Complex Number, **z**, has the following:

- A length or **Modulus,** $|z| = \operatorname{mod} z = r$
- An angle or **Argument,** $\theta = \arg z$
- A horizontal **Real Part,** $a = \operatorname{Re} z$
- A vertical **Imaginary Part,** $b = \operatorname{Im} z$

Importantly, the imaginary part of a complex number is <u>only</u> the coefficient of i.

Representations of ℂ-numbers

$$z \quad = \underset{rectangular}{a+bi} \quad = \underset{polar}{r\angle\theta} \quad = \underset{trigonometric}{r(\cos\theta + i\sin\theta)} \quad = \underset{exponential}{re^{i\theta}}$$

The 4 common forms of complex numbers are:

- **Rectangular**
- **Polar**

Requires a and b

- **Trigonometric**
- **Exponential**

Requires r and θ

Only the exponential form must have the angle in rads.!

Representations of $\mathbb{C}$-Numbers

Forms	Mathematical Model $z \in \mathbb{C}, \quad a, b, r \in \mathbb{R}$
Rectangular	$z = a + bi$
Polar	$z = \|z\| \angle \theta$
Trigonometric	$z = r(\cos\theta + i\sin\theta)$
Exponential	$z = re^{i\theta}$

Representations of ℂ-Numbers

Forms	Examples $z \in \mathbb{C}, \quad a, b, r \in \mathbb{R}$
Rectangular	$z = -2 + 5i$
Polar	$z = 55\angle 30^\circ$
Trigonometric	$z = 12(\cos 37^\circ + i \sin 37^\circ)$
Exponential	$z = 26e^{i\pi/6}$

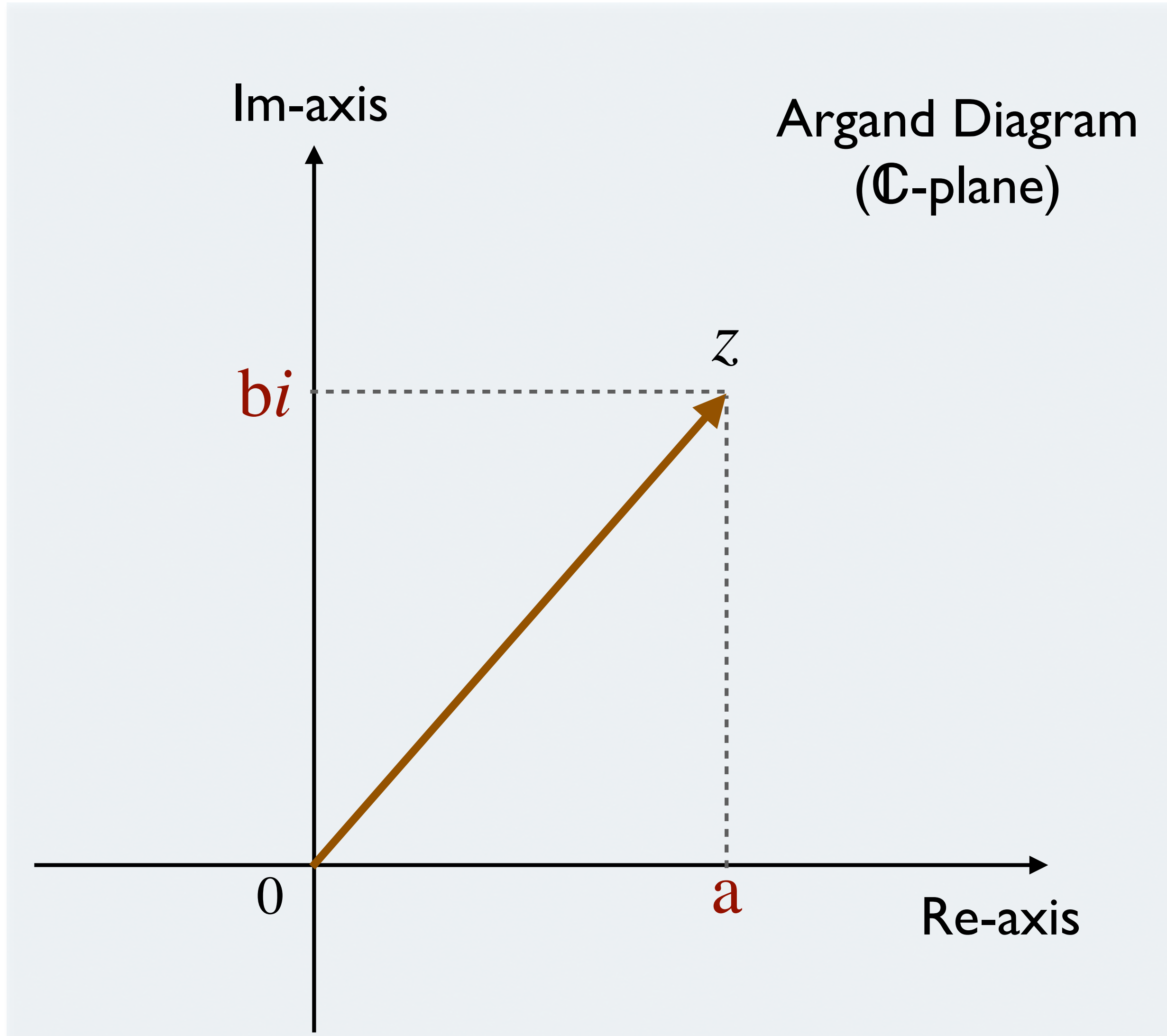
Representations of ℂ-Numbers

Forms	Examples	Components
Rect.	$z = -2 + 5i$	$a = -2; \quad b = 5$
Polar	$z = 55\angle 30^\circ$	$\lvert z \rvert = 55; \quad \theta = 30^\circ$
Trig.	$z = 12(\cos 37^\circ + i \sin 37^\circ)$	$\lvert z \rvert = 12; \quad \theta = 37^\circ$
Exp.	$z = 26e^{i\pi/6}$	$\lvert z \rvert = 26; \quad \theta = \frac{\pi}{6}$

Representations of ℂ-Numbers

RECTANGULAR

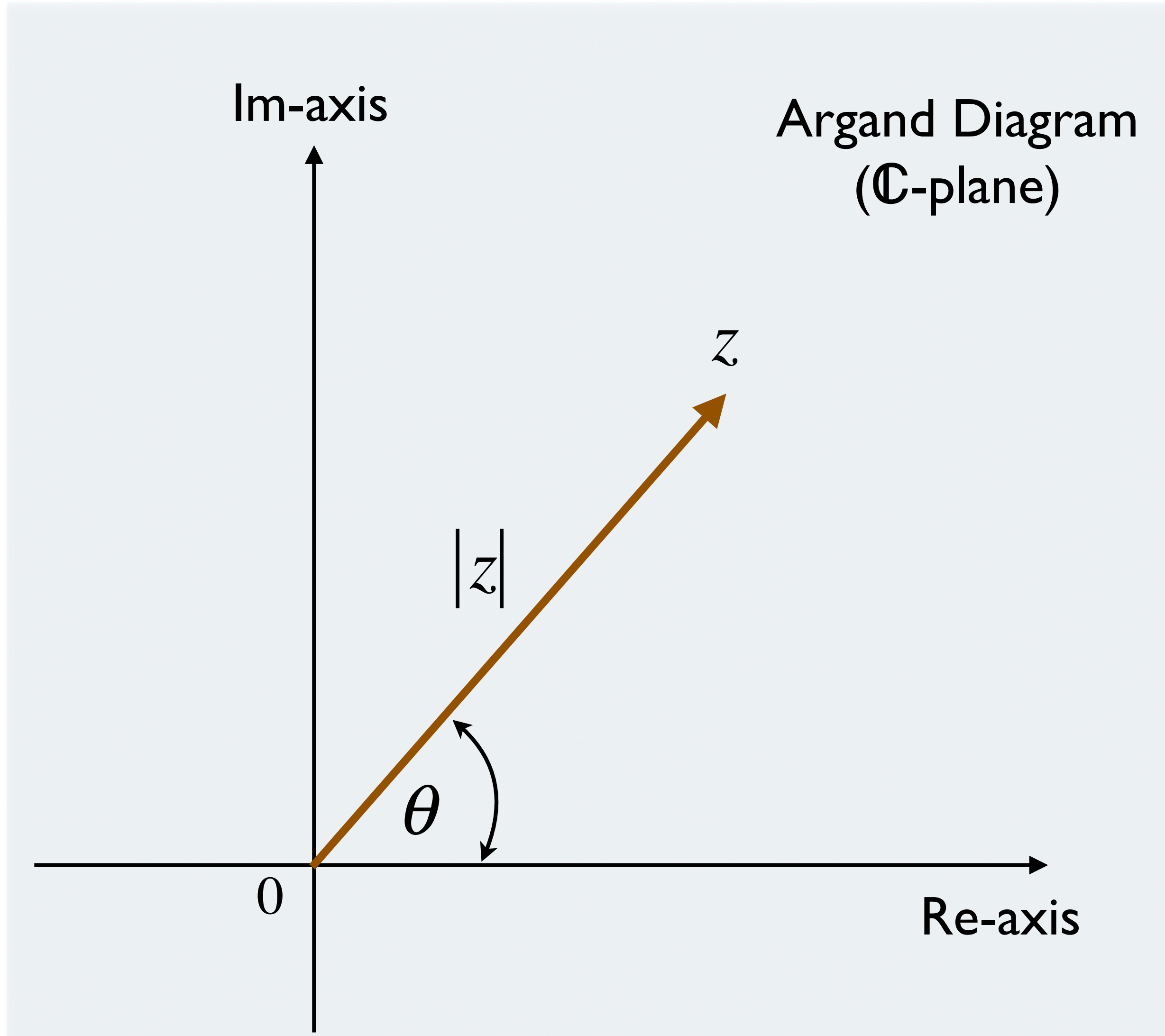
$z = a + bi$



Representations of ℂ-Numbers

POLAR

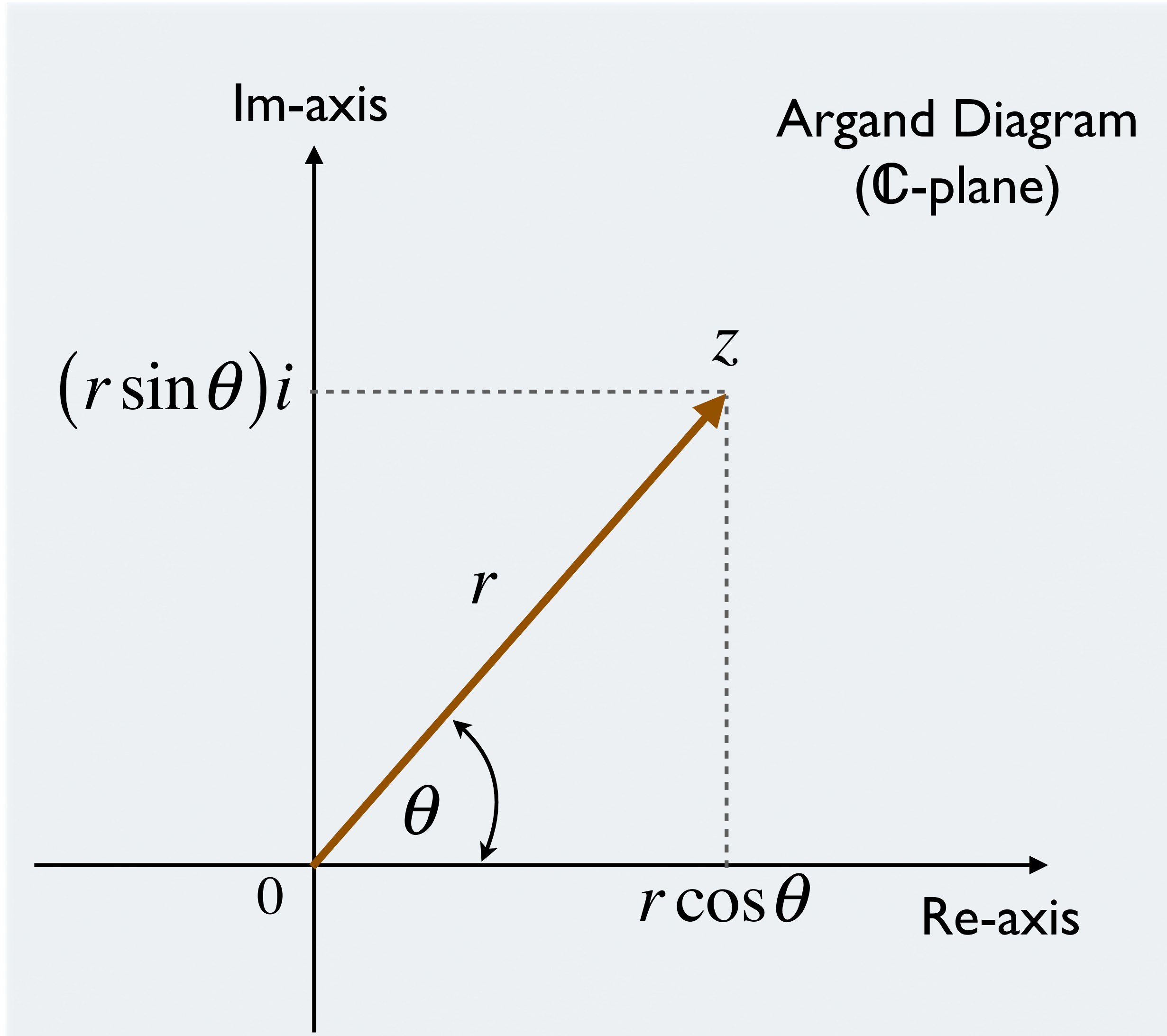
$$z = |z| \angle \theta$$



Representations of ℂ-Numbers

TRIGONOMETRIC

$$z = r(\cos\theta + i\sin\theta)$$

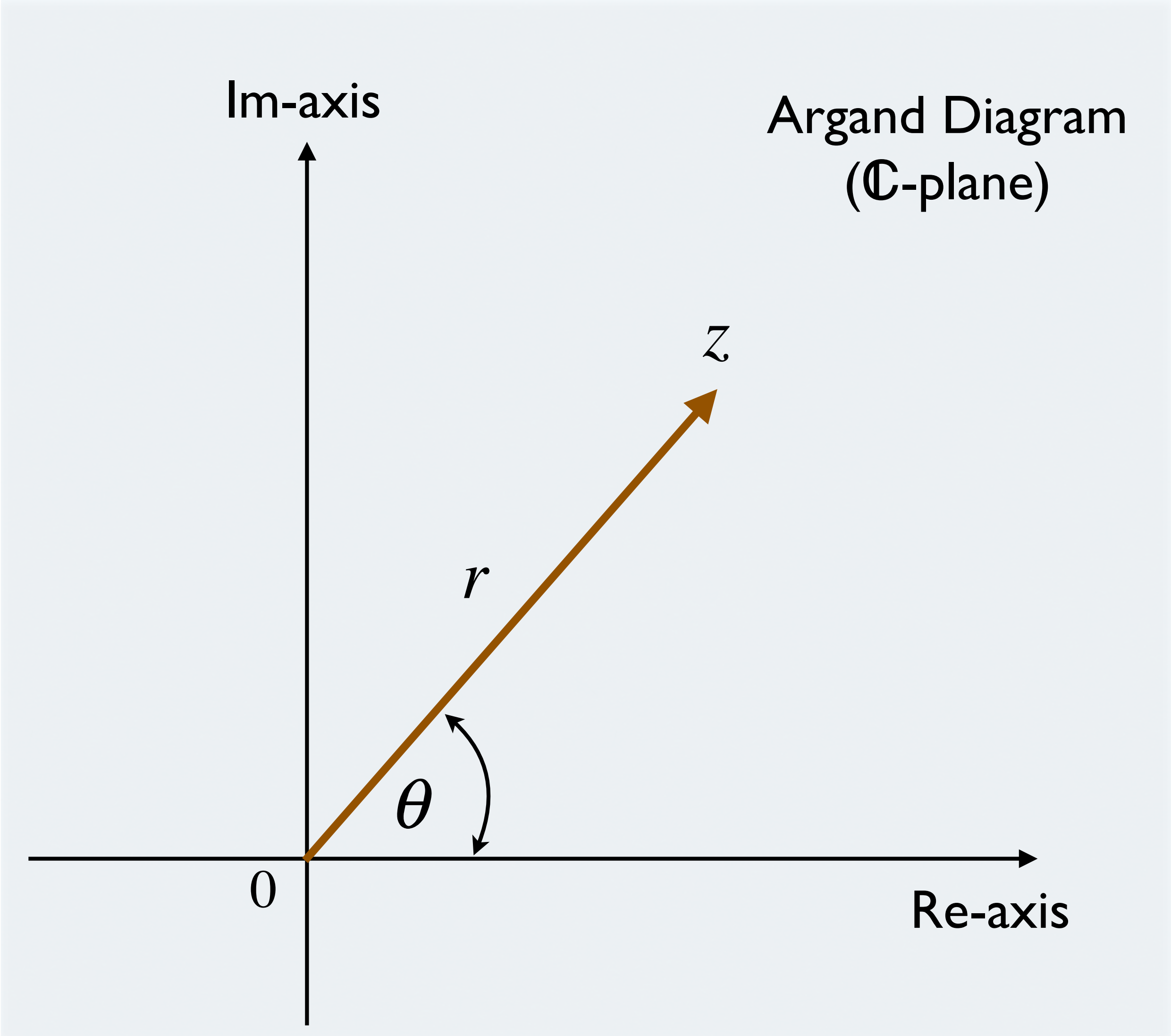


Representations of ℂ-Numbers

EXPONENTIAL

$$z = re^{i\theta}$$

Generally, in this form the angle, θ is in radians!



Complex Conjugate, z*

Representation of ℂ-numbers

Forms	Complex Number	Complex Conjugate
Rectangular	$z = a + bi$	$z^* = a - bi$
Polar	$z = \lvert z \rvert \angle \theta$	$z^* = \lvert z \rvert \angle -\theta$
Trigonometric	$z = r(\cos\theta + i\sin\theta)$	$z^* = r(\cos\theta - i\sin\theta)$
Exponential	$z = re^{i\theta}$	$z^* = re^{-i\theta}$

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Conversion of ℂ-numbers between Forms

$z \in \mathbb{C}$

Rectangular, Polar, Trigonometric or Exponential form.

$\rightleftharpoons$

$r = |z|$ $\quad \arg(z)$

$\mathrm{Re}(z)$ $\quad \mathrm{Im}(z)$

- Every Complex Number in any form can be **decomposed** into 4 parts.
- Every Complex Number can be **reconstructed** if at least 2 of its 4 parts are known!

Very Important Formulations!

Conversion of $\mathbb{C}$-numbers

(From one form to another)

$$z \underset{\text{rectangular}}{= a + bi} \quad \underset{\text{polar}}{= r\angle\theta} \quad \underset{\text{trigonometric}}{= r(\cos\theta + i\sin\theta)} \quad \underset{\text{exponential}}{= re^{i\theta}}$$

Decompositions	Formulations $z \in \mathbb{C}, \quad a,b,r \in \mathbb{R}$
Modulus	$\operatorname{mod} z = \lvert z \rvert = r = \sqrt{a^2 + b^2}$
Argument	$\arg z = \theta = \tan^{-1}(b/a)$
Real Part	$\operatorname{Re} z = a = \lvert z \rvert \cos\theta = r\cos\theta$
Imaginary Part	$\operatorname{Im} z = b = \lvert z \rvert \sin\theta = r\sin\theta$

Complex Numbers

Thank You

For more information, please visit:
http://www.boszik.net/mse