



CARIBBEAN EXAMINATIONS COUNCIL

**CSEC<sup>®</sup>**

# **Additional Mathematics**

**SYLLABUS  
SPECIMEN PAPER  
MARK SCHEME  
SUBJECT REPORTS**

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## **Additional Mathematics**

This Additional Mathematics course provides a variety of topics with related attributes which would enable Caribbean students to reason logically using the prior knowledge gained from the CSEC General Proficiency Mathematics. Candidates are expected to enter this course of study with a solid foundation of algebraic knowledge and mathematical reasoning.

On completing this course students will be able to make a smooth transition to higher levels of study in Mathematics, or move on to career choices where a deeper knowledge of the general concepts of Mathematics is required. This course of study, which includes fundamentals of Pure and Applied Mathematics, caters to diverse interests enabling students to develop critical-thinking skills applicable to other subject areas. This course thus provides insight into the exciting world of advanced mathematics, thereby equipping students with the tools necessary to approach any mathematical situation with confidence.

The syllabus is arranged as a set of topics, arranged in four sections as follows:

- Section 1 Algebra and Functions
- Section 2 Coordinate Geometry and Trigonometry
- Section 3 Introductory Calculus
- Section 4 Basic Mathematical Applications



**CARIBBEAN EXAMINATIONS COUNCIL**

**Caribbean Secondary Education Certificate  
CSEC®**

**ADDITIONAL MATHEMATICS  
SYLLABUS**

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# Additional Mathematics Syllabus

## ◆ RATIONALE

The Caribbean, with few limited resources, has prided itself in being a knowledge-based society utilizing the skills and assets of people, our greatest resource, to progress in a dynamic world where self-reliance is now more than ever a needed goal. Although different languages are spoken in the Caribbean, the language of Mathematics is one of the forms in which people of the Caribbean effectively communicate with each other.

This Additional Mathematics course provides a variety of topics with related attributes which would enable Caribbean students to reason logically using the prior knowledge gained from the CSEC General Proficiency Mathematics. Candidates are expected to enter this course of study with a solid foundation of algebraic knowledge and mathematical reasoning.

On completing this course students will be able to make a smooth transition to higher levels of study in Mathematics, or move on to career choices where a deeper knowledge of the general concepts of Mathematics is required. This course of study, which includes fundamentals of Pure and Applied Mathematics, caters to diverse interests enabling students to develop critical-thinking skills applicable to other subject areas.

Some of the underlying concepts of Mathematics will be explored to a level which fosters a deeper understanding and greater appreciation of Mathematics. This will give students the confidence and ability to approach problem-solving in enlightened ways and lead to creative methods of solving complex real-world problems.

This course thus provides insight into the exciting world of advanced mathematics, thereby equipping students with the tools necessary to approach any mathematical situation with confidence.

*This syllabus will contribute to the development of the Ideal Caribbean Person as articulated by the CARICOM Heads of Government in the following areas: “demonstrate multiple literacies, independent and critical thinking and innovative application of science and technology to problem solving. Such a person should also demonstrate a positive work attitude and value and display creative imagination and entrepreneurship”. In keeping with the UNESCO Pillars of Learning, on completion of this course the study, students will learn to do, learn to be and learn to transform themselves and society.*

## ◆ AIMS

The syllabus aims to:

1. build upon those foundational concepts, techniques and skills acquired at the CSEC General Proficiency Level and form linkages to areas of study at the Advanced Proficiency Level;
2. enhance ways of learning Mathematics;



3. stimulate further curiosity and analytical thinking in deriving solutions to problems which may not necessarily be solved by a single/unique approach;
4. promote effective mathematical communication;
5. develop abilities to reason logically;
6. develop skills in formulating real-world problems into mathematical form;
7. develop positive intrinsic mathematical values, such as, accuracy and rigour;
8. connect Mathematics with other disciplines such as Science, Business and the Arts.

## ◆ PRE-REQUISITES OF THE SYLLABUS

Any person with a good grasp of the contents of the syllabus of the Caribbean Secondary Education Certificate (CSEC) General Proficiency course in Mathematics, or equivalent, should be able to undertake this course. However, successful participation in this course will also depend critically on the possession of good verbal and written communication skills.

## ◆ ORGANIZATION OF THE SYLLABUS

The syllabus is arranged as a set of topics, and each topic is defined by its specific objectives and content. It is expected that students would be able to master the specific objectives and related content after successfully pursuing a course in Mathematics during five years of secondary education.

The topics are arranged in four sections as follows:

- |           |   |                                      |
|-----------|---|--------------------------------------|
| Section 1 | - | Algebra and Functions                |
| Section 2 | - | Coordinate Geometry and Trigonometry |
| Section 3 | - | Introductory Calculus                |
| Section 4 | - | Basic Mathematical Applications      |

## ◆ SUGGESTIONS FOR TEACHING THE SYLLABUS

For students who complete CSEC Mathematics in the fourth form year, Additional Mathematics can be done in the fifth form year. Alternatively students may begin Additional Mathematics in the fourth form and sit both CSEC Mathematics and Additional Mathematics examinations at the end of form five. Students may even do the CSEC Additional Mathematics as an extra subject simultaneously with CAPE Unit 1 in the Sixth Form.

## ◆ CERTIFICATION AND DEFINITION OF PROFILES

The syllabus will be examined for certification at the General Proficiency Level.

In addition to the overall grade, there will be a profile report on the candidate's performance under the following headings:

- (i) Conceptual Knowledge (CK);
- (ii) Algorithmic Knowledge (AK);
- (iii) Reasoning (R).

The assessment will test candidates' skills and abilities in relation to three cognitive levels.

- Conceptual knowledge - the ability to **recall**, **select** and **use** appropriate facts, concepts and principles in a variety of contexts.
- Algorithmic knowledge - the ability to **manipulate** mathematical expressions and procedures using appropriate symbols and language, logical deduction and inferences.
- Reasoning - the ability to **select**, **use** and **evaluate** mathematical models and **interpret** the results of a mathematical solution in terms of a given real-world problem, and to **engage** problem-solving.

## ◆ FORMAT OF THE EXAMINATIONS

The examination will consist of three papers: Paper 01, an objective type paper, Paper 02, an essay or problem-solving type paper and Paper 03, the School Based Assessment which will focus on investigation or problem solving related to any area of the syllabus.

**Paper 01** (1 hour 30 minutes) This Paper will consist of 45 multiple-choice items, sampling the Core as follows:

Section	Topics	No. of items	Total
1	Algebraic Operations	2	20
	Quadratics	3	
	Inequalities	2	
	Functions	4	
	Surds, Indices and Logarithms	5	
	Series	4	
2	Co-ordinate Geometry	3	14
	Vectors	3	
	Trigonometry	8	
3	Differentiation	5	11
	Integration	6	
<b>Total</b>			<b>45</b>

The 45 marks will be weighted to 60 marks

**Paper 02**  
(2 hours 40 minutes)

This Paper will consist of two sections, Section I and Section II.

Section I: 80 marks  
This section will consist of 6 compulsory structured and problem-solving type questions based on Sections 1, 2 and 3 of the syllabus: Algebra and Functions; Coordinate Geometry, Vectors and Trigonometry; and Introductory Calculus.

Section II: 20 marks  
This section will consist of 2 structured or problem-solving questions based on Section 4 of the syllabus, Basic Mathematical Applications. One question will be set on Data Representation and Probability and the other question will be set on Kinematics. Candidates will be required to answer only **ONE** question from this section. Each question will be allocated 20 marks.

The marks allocated to the sections are shown below.

Sections		No. of questions	Marks			Total
			CK	AK	R	
1	Algebra and Functions	2	6	12	10	28
2	Coordinate Geometry, Vectors and Trigonometry	2	6	10	8	24
3	Introductory Calculus	2	6	12	10	28
4	Basic Mathematical Applications	1 of 2	6	8	6	20
<b>Total Marks</b>			24	42	34	100

## SCHOOL BASED ASSESSMENT

### Paper 03/1

This paper comprises a project requiring candidates to demonstrate the practical application of Mathematics in everyday life. In essence it should allow candidates to probe, describe and explain a mathematical area of interest and communicate the findings using mathematical symbols, language and tools. The topic(s) chosen may be from any section or combination of different sections of the syllabus.



The project may require candidates to collect data, or may be theory based, requiring solution or proof of a chosen problem.

The total marks for Paper 03/1 is 20 and will contribute 20% toward the final assessment. See Guidelines for School Based Assessment on pages 29 – 46.

**Paper 03/2 (Alternative to Paper 03/1), examined externally.**

This paper is an alternative to Paper 03/1 and is intended for private candidates. This paper comprises one question. The given topic(s) may be from any section or combination of different sections of the syllabus. The duration of the paper is 1 ½ hours.

**WEIGHTING OF PAPER AND PROFILES**

The percentage weighting of the examination components and profiles is as follows:

**Table 1 – Percentage Weighting of Papers and Profiles**

PROFILES	PAPER 01	PAPER 02	PAPER 03	TOTAL %
Conceptual (CK)	12 (16)	24	04 (08)	47 (24%)
Algorithmic Knowledge (AK)	24 (32)	42	06 (12)	87 (44%)
Reasoning (R)	09 (12)	34	10 (20)	66 (32%)
TOTAL	45 (60)	100	20 (40)	200
[%]	30%	50%	20%	100%

**◆ REGULATIONS FOR RESIT CANDIDATES**

1. Resit candidates must complete Papers 01 and 02 of the examination for the year for which they re-register. Resit candidates who have earned at least 50% of the **MODERATED** score for the SBA component may elect not to repeat this component, provided they re-write the examination no later than the year following their first attempt. The scores for the SBA can be transferred once only, that is, to the examination immediately following that for which they were obtained.
2. Resit candidates who have obtained less than 50% of the **MODERATED** scores for the SBA component must repeat the component at any subsequent sitting.
3. Resit candidates must be entered through a school or other approved educational institution.

## ◆ REGULATIONS FOR PRIVATE CANDIDATES

Candidates who are registered privately will be required to sit Paper 01, Paper 02 and Paper 03/2.

Private candidates must be entered through institutions recognized by the Council.

## ◆ MISCELLANEOUS SYMBOLS

=	is equal to
≠	is not equal to
<	is less than
≤	is less than or equal to (is not greater than)
>	is greater than
≥	is greater than or equal to (is not less than)
≡	is identical to
≈	is approximately equal to
∝	is proportional to
∞	infinity

### Operations

$$\sum_{i=1}^n x_i \quad x_1 + x_2 + x_3 + \dots + x_n$$

### Functions

P	the set of Real Numbers
$f(x)$	the value of the function $f$ at $x$
$f^{-1}$	the inverse function of the function $f$
$g * f, gf$	the composite function $f$ and $g$ which is defined by $(g * f)(x)$ or $gf(x) = g[f(x)]$
$\frac{dy}{dx}, y'$	the first derivative of $y$ with respect to $x$
$\frac{d^n y}{dx^n}, y^n$	the $n^{\text{th}}$ derivative of $y$ with respect to $x$
$f'(x), f''(x), \dots,$	the first, second, ..., $n^{\text{th}}$ derivatives of $f(x)$ with respect to $x$
$f^{(n)}(x)$	
$\dot{x}, \ddot{x}$	the first and second derivatives of $x$ with respect to time
$\lg x$	the logarithm of $x$ to base 10
$\int y \, dx$	the indefinite integral of $y$ with respect to $x$
$\int_a^b y \, dx$	the definite integral of $y$ with respect to $x$ between the limits $x = a$ and $x = b$

## Probability and Statistics

$A \cap B$	union of the events $A$ and $B$
$A \cup B$	intersection of the events $A$ and $B$
$S$	the possibility sample space
$P(A)$	the probability of the event $A$ occurring
$P(\bar{A})$	the probability of the event $A$ not occurring
$P(A B)$	the conditional probability of the event $A$ occurring given the event $B$ has occurred.

## Vectors

$\underline{a}, \mathbf{a}$	the vector $\mathbf{a}$
$\overrightarrow{AB}$	the vector represented in magnitude and direction by the directed line segment $AB$
$ \overrightarrow{AB} $	the magnitude of $\overrightarrow{AB}$
$\hat{\mathbf{a}}$	a unit vector in the direction of $\mathbf{a}$
$ \mathbf{a} $	the magnitude of $\mathbf{a}$
$\mathbf{a} \cdot \mathbf{b}$	the scalar (dot) product of $\mathbf{a}$ and $\mathbf{b}$
$\mathbf{i}, \mathbf{j}$	unit vectors in the direction of the Cartesian coordinate axes, $x$ and $y$ respectively
$\begin{pmatrix} x \\ y \end{pmatrix}$	$x\mathbf{i} + y\mathbf{j}$

## Mechanics

$x$	displacement
$v, \dot{\mathbf{x}}$	velocity
$a, \dot{\mathbf{v}}, \ddot{\mathbf{x}}$	acceleration
$g$	acceleration due to gravity

## ◆ LIST OF FORMULAE

### Arithmetic Series

$$T_n = a + (n-1)d \quad S_n = \frac{n}{2}[2a + (n-1)d]$$

### Geometric Series

$$T_n = ar^{n-1} \quad S_n = \frac{a(r^n - 1)}{r - 1} \quad S_\infty = \frac{a}{1 - r}, \quad -1 < r < 1 \text{ or } |r| < 1$$

Circle:  $x^2 + y^2 + 2fx + 2gy + c = 0 \quad (x + f)^2 + (y + g)^2 = r^2$

### Vectors

$$\hat{\mathbf{v}} = \frac{\mathbf{v}}{|\mathbf{v}|} \quad \cos \theta = \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}| |\mathbf{b}|} \quad |\mathbf{v}| = \sqrt{(x^2 + y^2)} \text{ where } \mathbf{v} = xi + yj$$

### Trigonometry

$$\sin(A \pm B) \equiv \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) \equiv \cos A \cos B \mp \sin A \sin B$$

### Differentiation

$$\frac{d}{dx} (ax + b)^n = an(ax + b)^{n-1}$$

$$\frac{d}{dx} \sin x = \cos x$$

$$\frac{d}{dx} \cos x = -\sin x$$



### Statistics

$$\bar{x} = \frac{\sum_{i=1}^n x_i}{n} = \frac{\sum_{i=1}^n f_i x_i}{\sum_{i=1}^n f_i} \quad S^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n} = \frac{\sum_{i=1}^n f_i x_i^2}{\sum_{i=1}^n f_i} - (\bar{x})^2$$

### Probability

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

### Kinematics

$$v = u + at \quad v^2 = u^2 + 2as \quad s = ut + \frac{1}{2}at^2$$

## ◆ USE OF ELECTRONIC CALCULATORS

Candidates are expected to have an electronic non-programmable calculator and are encouraged to use such a calculator in Paper 02. Candidates will also be allowed to use a calculator in Papers 01 and 03.

Guidelines for the use of electronic calculators are listed below.

1. Silent, electronic hand-held calculators may be used.
2. Calculators should be battery or solar powered.
3. Candidates are responsible for ensuring that calculators are in working condition.
4. Candidates are permitted to bring a set of spare batteries in the examination room.
5. **No** compensation will be given to candidates because of faulty calculators.
6. **No** help or advice is permitted on the use or repair of calculators during the examination.
7. Sharing calculators is **not** permitted in the examination room.
8. Instruction manuals, and external storage media (for example, card, tape, disk, smartcard or plug-in modules) are **not** permitted in the examination room.
9. Calculators with graphical display, data bank, dictionary or language translation are **not** allowed.
10. Calculators that have the capability of communication with any agency in or outside of the examination room are **prohibited**.

## ◆ SECTION 1 : ALGEBRA AND FUNCTIONS

### GENERAL OBJECTIVES

On completion of this Section, students should:

1. be confident in the manipulation of algebraic expressions and the solutions of equations and inequalities;
2. understand the difference between a sequence and a series;
3. distinguish between convergence and divergence of arithmetic and geometric series;
4. understand the concept of a function;
5. develop the ability to use concepts to model and solve real-world problems.

### SPECIFIC OBJECTIVES

### CONTENT

#### A. Algebra

Students should be able to:

- |   |   |
|---|---|
| 1. perform operations of addition, subtraction, multiplication and division of polynomial and rational expressions; | Addition, subtraction, multiplication, division and factorization of algebraic expressions.     |
| 2. factorize polynomial expressions, of degree less than or equal to 4, leading to real linear factors;             | Division of a polynomial of degree less than or equal to 4 by a linear or quadratic polynomial. |
| 3. apply the Remainder Theorem;   | Remainder Theorem.  |
| 4. use the Factor Theorem to find factors and to evaluate unknown coefficients.                                     | Factor Theorem.   |

#### B. Quadratics

Students should be able to:

- |   |                                     |
|---|-------------------------------------|
| 1. express the quadratic function $ax^2 + bx + c = 0$ in the form | Quadratic equations in one unknown. |
|---|-------------------------------------|

## ALGEBRA AND FUNCTIONS (cont'd)

### SPECIFIC OBJECTIVES

Students should be able to:

$a(x + h)^2 + k = 0$ , where  $h$  and  $k$  are constants to be determined.

- determine maximum or minimum values and range of quadratic functions by completion of the square;
- sketch the graph of the quadratic function, including maximum or minimum points;
- determine the nature of the roots of a quadratic equation;
- solve equations in  $x$  reducible to a quadratic equation, for example,  
 $x^4 - 6x^2 + 8 = 0$  and  
 $x - 2\sqrt{x} + 1 = 0$  ;
- use the relationship between the sums and products of the roots and the coefficients of  $ax^2 + bx + c = 0$ ;
- solve two simultaneous equations in 2 unknowns in which one equation is linear and the other equation is quadratic.

### C. Inequalities

Students should be able to:

- find the solution sets of quadratic inequalities using algebraic and graphical methods;
- find the solution sets of inequalities

### CONTENT

Completing the square.

Graphs of quadratic functions.

Applications of sums and products of the roots of quadratic equations.

Solution of equations (one linear and one quadratic).

Quadratic inequalities in one unknown.

Rational inequalities with linear factors.

## ALGEBRA AND FUNCTIONS (cont'd)

### SPECIFIC OBJECTIVES

of the form  $\frac{ax+b}{cx+d} > 0; \geq 0; < 0; \leq 0$  using algebraic and graphical methods.

#### D. Functions

Students should be able to:

1. use terms related to functions;
2. determine the range of a function given its domain;
3. determine whether a given function is many-to-one or one-to-one;
4. determine the inverse of a given function, (if it exists);
5. plot and sketch functions and their inverses, (if they exist);
6. state the geometrical relationship between the function  $y = f(x)$  and its inverse  $f^{-1}(x)$ ;
7. find the composition of two functions;
8. recognize that, if  $g$  is the inverse of  $f$ , then  $f[g(x)] = x$ , for all  $x$ , in the domain of  $g$ .

#### E. Surds, Indices, and Logarithms

Students should be able to:

1. perform operations involving surds;

### CONTENT

Arrow diagrams. Function, domain, co-domain, range, open interval, half open interval, closed interval, one-to-one function, onto function, one-to-one correspondence, inverse and composition of functions;

Rational and polynomial functions up to degree less than or equal to 3.

Graphical methods and horizontal line test. Formal proof not required.

Exclude rational functions.

$f^{-1}(x)$  as the reflection of  $f(x)$  in the line  $y = x$ .

Addition, subtraction, multiplication and rationalization of denominators of surds.



## ALGEBRA AND FUNCTIONS (cont'd)

### SPECIFIC OBJECTIVES

Students should be able to:

2. use the laws of indices to solve exponential equations with one unknown;
3. use the fact that  $\log_a b = c \Leftrightarrow a^c = b$  where  $a$  is any positive whole number;
4. simplify expressions by using *the laws*:

(i)  $\log_a (PQ) = \log_a P + \log_a Q$ ;

(ii)  $\log_a \left( \frac{P}{Q} \right) = \log_a P - \log_a Q$ ;

(iii)  $\log_a P^b = b \log_a P$ ;

(iv)  $\log_a a = 1$ ;

(v)  $\log_a 1 = 0$ ;

5. solve logarithmic equations;
6. use logarithms to solve equations of the form  $a^x = b$ ;
7. apply logarithms to problems involving the transformation of a given relationship to linear form.

### F. Sequences and Series

Students should be able to:

1. define a sequence of terms  $\{a_n\}$  where  $n$  is a positive integer;
2. write a specific term from the formula for the  $n^{\text{th}}$  term of a sequence;

### CONTENT

Equations reducible to linear and quadratic forms.

The relationship between indices and logarithms.

Laws of logarithms.

Example,

$$\log_a (2x + 5) - \log_a (3x - 10) = \log_a (x - 14).$$

Linear Reduction.

## ALGEBRA AND FUNCTIONS (cont'd)

### SPECIFIC OBJECTIVES

3. use the summation ( $\Sigma$ ) notation;
4. define a series, as the sum of the terms of a sequence;
5. identify arithmetic and geometric series;

### CONTENT

Series as the sum of the terms of a sequence.

Students should be able to:

6. obtain expressions for the general terms and sums of finite arithmetic and finite and infinite geometric series;
7. show that all arithmetic series (except for zero common difference) are divergent, and that geometric series are convergent only if  $-1 < r < 1$ , where  $r$  is the common ratio;
8. calculate the sum of arithmetic series to a given number of terms;
9. calculate the sum of geometric series to a given number of terms;
10. find the sum of a convergent geometric series.

The sums of finite arithmetic, and finite and infinite geometric series.

### Suggested Teaching and Learning Activities

To facilitate students' attainment of the objectives of this Section, teachers are advised to engage students in the teaching and learning activities listed below.

#### Number Systems

Teachers should do a brief review of the Number Systems before starting Algebra and Functions.



### Functions (one-to-one, onto, one-to-one correspondence) – Inverse Function

Students should explore the mapping properties of quadratic functions which:

- (i) will, or will not, be one-to-one, depending on which subset of the real line is chosen as the domain;
- (ii) will be onto, if the range is taken as the co-domain (completion of the square is useful here);
- (iii) if both one-to-one and onto, will have an inverse function which can be obtained by solving a quadratic equation.

Example: Use the function  $f : A \rightarrow B$  given by  $f(x) = 3x^2 + 6x + 5$ , where the domain  $A$  is alternatively the whole of the real line, or the set  $\{x \in \mathbf{R} \mid x \geq -1\}$ , and the co-domain  $B$  is  $\mathbf{R}$  or the set  $\{y \in \mathbf{R} \mid y \geq 2\}$ .

### Series

Teachers should apply the concepts of the arithmetic and geometric series to solve real-world problems such as investments.

## ◆ SECTION 2: COORDINATE GEOMETRY, VECTORS AND TRIGONOMETRY

### GENERAL OBJECTIVES

On completion of this Section, students should:

1. develop the ability to represent and deal with points in the coordinate plane through the use of geometry and vectors;
2. develop the ability to manipulate and describe the behaviour of trigonometric functions;
3. develop skills to solve trigonometric equations;
4. develop skills to prove simple trigonometric identities;
5. develop the ability to use concepts to model and solve real-world problems.

### SPECIFIC OBJECTIVES

### CONTENT

#### A. Coordinate Geometry

Students should be able to:

- |  |  |
|--|--|
| 1. find the equation of a straight line;   | The gradient of a line segment.  |
| 2. determine whether lines are parallel or mutually perpendicular using the gradients; | Relationships between the gradients of parallel and mutually perpendicular lines.  |
| 3. find the point of intersection of two lines;  |  |
| 4. write the equation of a circle;   | The equation of the circle in the forms<br>$(x + f)^2 + (y + g)^2 = r^2$ ,<br>$x^2 + y^2 + 2fx + 2gy + c = 0$ ,<br>where $a, b, f, g, c, r \in \mathbb{P}$ . |
| 5. find the centre and radius of a given circle;                                       |  |
| 6. find equations of tangents and normals at given points on circles;                  | Tangents and normals to the circle.  |
| 7. find the points of intersection of a curve with a straight line.                    |  |



## SECTION 2: COORDINATE GEOMETRY, VECTORS AND TRIGONOMETRY (cont'd)

SPECIFIC OBJECTIVES	CONTENT
<b>B. Vectors</b>	
Students should be able to:	
1. express a vector in the form $\begin{pmatrix} x \\ y \end{pmatrix}$ or $x\mathbf{i} + y\mathbf{j}$ ; $x, y \in \mathbb{P}$ ;	Two-dimensional vectors and their geometric representations.
2. define equal vectors;	Equality of vectors.
3. add and subtract vectors;	
4. multiply a vector by a scalar quantity;	
5. derive and use unit vectors;	Unit vectors.
6. find displacement vectors;	Position and displacement vectors.
7. find the magnitude and direction of a vector;	Modulus and direction of a vector.
8. define the scalar product of two vectors:	Scalar (dot) product of 2 vectors.
(i) in terms of their components;	
(ii) in terms of their magnitudes and the angle between them;	
9. find the angle between two given vectors;	
10. apply properties of parallel and perpendicular vectors.	Problems involving parallel and perpendicular vectors.
<b>C. Trigonometry</b>	
<i>(All angles will be assumed to be measured in radians unless otherwise stated)</i>	
1. define the radian;	
2. convert degrees to radians and radians to degrees;	
3. use the formulae for arc length $l = rq$ and sector area $A = \frac{1}{2} r^2 q$ ;	Applications of arc length and sector area.

## SECTION 2: COORDINATE GEOMETRY, VECTORS AND TRIGONOMETRY (cont'd)

SPECIFIC OBJECTIVES	CONTENT
Students should be able to:	
4. evaluate sine, cosine and tangent for angles of any size given either in degrees or radians;	
5. evaluate the exact values of sine, cosine and tangent for $q = 0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}, \boxed{\times}, \boxed{X}, 2\pi;$	Include related angles such as $\frac{2\pi}{3}, \frac{4\pi}{3}$ .
6. graph the functions $\sin kx, \cos kx, \tan kx$ , where $k$ is 1 or 2 and $0 \leq x \leq 2\pi$ ;	
7. derive the identity $\cos^2 \theta + \sin^2 \theta \equiv 1$ ;	
8. use the formulae for $\sin (A \pm B)$ , $\cos (A \pm B)$ and $\tan (A \pm B)$ ;	Compound-angle formulae.
9. derive the multiple angle identities for $\sin 2x, \cos 2x, \tan 2x$ ;	Double-angle formulae.
10. use Specific Objectives 7, 8 and 9 above to prove simple identities;	
11. find solutions of simple trigonometric equations for a given range, including those involving the use of $\cos^2 q + \sin^2 q \equiv 1$ .	Solution of simple trigonometric equations including graphical interpretation but excluding general solution.

### Suggested Teaching and Learning Activities

To facilitate students' attainment of the objectives of this Section, teachers are advised to engage students in the teaching and learning activities listed below.

## Trigonometric Identities

Teachers should derive the trigonometric identities and formulae where appropriate. However, students are not expected to know the proofs of the following trigonometric formulae:

$$\sin (A \pm B), \cos (A \pm B), \tan (A \pm B).$$

Students should recognise that  $\tan \theta \equiv \frac{\sin \theta}{\cos \theta}$ .

Students should use the equilateral and the isosceles right angled triangle to derive the exact values of **sine, cosine and tangent of**  $\left(\frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}\right)$ .

Students should also derive the trigonometric functions  $\sin x$  and  $\cos x$  for angles  $x$  of any value (including negative values), using the coordinates of points on the unit circle.

Students should be made aware of the relationships between the unit circle and its quadrants and the related angles (principal and secondary solutions).

## ◆ SECTION 3: INTRODUCTORY CALCULUS

### GENERAL OBJECTIVES

On completion of this Section, students should:

1. understand the relationships between the derivative of a function at a point and the behaviour of the function and its tangent at that point;
2. be confident in differentiating and integrating given functions;
3. understand the relationship between integration and differentiation;
4. know how to calculate areas and volumes using integration;
5. develop the ability to use concepts to model and solve real-world problems.

### SPECIFIC OBJECTIVES

### CONTENT

#### A. Differentiation

Students should be able to:

- |  |   |
|--|---|
| 1. use the concept of the derivative at a point $x = c$ as the gradient of the tangent to the graph at $x = c$ ; | The gradient of a curve.                        |
| 2. define the derivative at a point as a limit;  | The derivative as a limit (intuitive approach). |
| 3. use the $f'(x)$ and $\frac{dy}{dx}$ notation for the first derivative of $f(x)$ ;                             |   |
| 4. use $\frac{d}{dx} x^n = n x^{n-1}$ where $n$ is any real number;  | The derivative of $x^n$ .                       |
| 5. use $\frac{d}{dx} \sin x = \cos x$<br>and $\frac{d}{dx} \cos x = -\sin x$ ;                                   | The derivatives of $\sin x$ and $\cos x$ .      |

### SECTION 3: INTRODUCTORY CALCULUS (cont'd)

SPECIFIC OBJECTIVES	CONTENT
Students should be able to:	
6. use simple rules of derivatives to find derivatives of sums and multiples of functions;	Simple rules of derivatives: $(i) \frac{d}{dx} c f(x) = c \frac{d}{dx} f(x) \text{ where } c \text{ is a constant}$ $(ii) \frac{d}{dx} f(x) \pm g(x) = \frac{d}{dx} f(x) \pm \frac{d}{dx} g(x)$
7. use Specific Objectives 4, 5 and 6 above to calculate derivatives of polynomials and trigonometric functions;	Differentiation of simple polynomials and trigonometric functions involving sine and cosine only.
8. apply the chain rule in the differentiation of composite functions;	Function of a function, the chain rule.
9. differentiate products and quotients of simple polynomials and trigonometric functions;	Product and quotient rules.
10. use the concept of the derivative as a rate of change;	
11. use the concept of stationary points;	Stationary points.
12. determine the nature of stationary points;	
13. locate stationary points, maxima and minima, by considering sign changes of the derivative;	Point(s) of inflexion not included.
14. calculate the second derivative, $f''(x)$ ;	Second derivatives of functions.
15. interpret the significance of the sign of the second derivative;	
16. use the sign of the second derivative to determine the nature of stationary points;	
17. obtain equations of tangents and normals to curves.	

### SECTION 3: INTRODUCTORY CALCULUS (cont'd)

SPECIFIC OBJECTIVES	CONTENT
<b>B. Integration</b>	
Students should be able to:	
1. recognize integration as the reverse process of differentiation;	Anti-derivatives.
2. use the notation $\int f(x) dx$ ;	Indefinite integrals (concept and use).
3. show that the indefinite integral represents a family of functions which differ by constants;	
4. use simple rules of integration;	Rules of Integration.  (i) $\int cf(x) dx = c \int f(x) dx$ , where $c$ is a constant;  (ii) $\int \{f(x) \pm g(x)\} dx = \int f(x) dx \pm \int g(x) dx$ .
5. integrate functions of the form $(ax \pm b)^n$ where $a, b, n$ are real and $n \neq -1$ ;	Integration of polynomials.
6. find indefinite integrals using formulae and integration theorems;	
7. integrate simple trigonometric functions;	Integration of $a \sin x \pm b \cos x$ , where $a$ and $b$ are constants.
8. compute definite integrals;	The definite integral:  $\int_a^b f(x) dx = F(b) - F(a)$ , where $F(x)$ is an indefinite integral of $f(x)$ .
9. formulate the equation of a curve given its gradient function and a point on the curve;	
10. apply integration to:	
(i) find the area of the region in the first quadrant bounded by a curve and the lines parallel to the y-axis;	

### SECTION 3: INTRODUCTORY CALCULUS (cont'd)

#### SPECIFIC OBJECTIVES

#### CONTENT

Students should be able to:

- (ii) find volumes of revolution about the  $x$ -axis, for polynomials up to and including degree 2.

The region of the curve to be rotated must be in the first quadrant only.

#### Suggested Teaching and Learning Activities

To facilitate students' attainment of the objectives of this Section, teachers are advised to engage students in the teaching and learning activities listed below.

#### Differentiation

Teachers should introduce the derivative as a limit but take only an intuitive approach at this stage using diagrams and not first principles.

A graphical explanation of  $\frac{d}{dx}(\sin x) = \cos x$  would suffice.

Teachers should explain the concept of increasing and decreasing functions, but it will not be tested.

Curve sketching using the differential calculus is not required.

#### The Area under the Graph of a Continuous Function

Class discussion should play a major role in dealing with this topic. Activities such as that which follows may be performed to motivate the discussion.

Example of classroom activity:

Consider a triangle of area equal to  $\frac{1}{2}$  unit<sup>2</sup>, bounded by the graphs of  $y = x$ ,  $y = 0$  and  $x = 1$ .

- (i) Sketch the graphs and identify the triangular region enclosed.
- (ii) Subdivide the interval  $[0, 1]$  into  $n$  equal subintervals.
- (iii) Evaluate the sum,  $s(n)$ , of the areas of the inscribed rectangles and  $S(n)$ , of the circumscribed rectangles, erected on each subinterval.
- (iv) By using different values of  $n$ , for example  $n = 5, 10, 15, 20$ , show that both  $s(n)$  and  $S(n)$  get closer to the area of the given region.

## ◆ SECTION 4: BASIC MATHEMATICAL APPLICATIONS

### GENERAL OBJECTIVES

On completion of this Section, students should:

1. appreciate that data can be represented both graphically and numerically to initiate analysis;
2. understand the concept of probability;
3. appreciate that probability models can be used to describe real world situations;
4. apply mathematical models to the motion of a particle.

### SPECIFIC OBJECTIVES

### CONTENT

#### A. Data Representation and Analysis

Students should be able to:

- |  |   |
|--|---|
| 1. distinguish between types of data;  | Qualitative and quantitative data, discrete and continuous data.  |
| 2. represent numerical data diagrammatically;  | Stem-and-leaf diagrams and box-and-whisker plots.   |
| 3. outline the relative advantages and disadvantages of stem-and-leaf diagrams and box-and-whisker plots in data analyses; |   |
| 4. interpret stem-and-leaf diagrams and box-and-whiskers plots;  |   |
| 5. determine quartiles and percentiles from raw data, grouped data, stem-and-leaf diagrams, box-and-whisker plots;         | Percentiles.  |
| 6. calculate measures of central tendency and dispersion;  | Mode, mean, median, range, interquartile range, semi-inter-quartile range, variance and standard deviation of ungrouped and grouped data; |
| 7. explain how the standard deviation measures the spread of a set of data.  |   |



## SECTION 4: BASIC MATHEMATICAL APPLICATIONS (cont'd)

SPECIFIC OBJECTIVES	CONTENT
<b>B. Probability Theory</b>	
Students should be able to:	
1. distinguish among the terms experiment, outcome, sample space and event;	Concept of probability.
2. calculate the probability of event $A$ , $P(A)$ , as the number of outcomes of $A$ divided by the total number of possible outcomes, when all outcomes are equally likely and the sample space is finite;	Classical probability. Relative frequency as an estimate of probability.
3. use the basic laws of probability:  (i) the sum of the probabilities of all the outcomes in a sample space is equal to one;  (ii) $0 \leq P(A) \leq 1$ for any event $A$ ;  (iii) $P(A') = 1 - P(A)$ , where $P(A')$ is the probability that event $A$ does not occur;	
4. use $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ to calculate probabilities;	The addition rule.
5. identify mutually exclusive events $A$ and $B$ such that $P(A \cap B) = 0$ ;	Mutually exclusive events.
6. calculate the conditional probability $P(A B)$ where $P(A B) = \frac{P(A \cap B)}{P(B)}$ ;	Conditional probability.
7. identify independent events;	Independent events.
8. use the property $P(A \cap B) = P(A) \times P(B)$ or $P(A B) = P(A)$ where $A$ and $B$ are independent events;	
9. construct and use possibility space diagrams, tree diagrams and Venn diagrams to solve problems involving probability.	Possibility space diagrams, tree diagrams and Venn diagrams.

## SECTION 4: BASIC MATHEMATICAL APPLICATIONS (cont'd)

SPECIFIC OBJECTIVES	CONTENT
<b>C. Kinematics of Motion along a straight line</b>	
Students should be able to:	
1. distinguish between distance and displacement, and speed and velocity;	Scalar and vector quantities.
2. draw and use displacement-time and velocity-time graphs;	Displacement-time graphs and velocity-time graphs.
3. calculate and use displacement, velocity, acceleration and time in simple equations representing the motion of a particle in a straight line;	Displacement, velocity and acceleration.
4. apply where appropriate the following rates of change:	Variable motion of a particle.
$v = \frac{dx}{dt} = \dot{x};$ $a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = \ddot{x}$ where $x, \dot{x}, \ddot{x}$ represent displacement, velocity and acceleration respectively (restricted to calculus from Section 3).	

### Suggested Teaching and Learning Activities

To facilitate students' attainment of the objectives of this Section, teachers are advised to engage students in the teaching and learning activities listed below. Whenever possible, class discussions and presentations should be encouraged.

#### Probability

Consider the three scenarios given below.

1. Throw two dice. Find the probability that the sum of the dots on the uppermost faces of the dice is 6.

2. An insurance salesman visits a household. What is the probability that he will be successful in selling a policy?
3. A hurricane is situated 500 km east of Barbados. What is the probability that it will hit the island?

These three scenarios are very different for the calculation of probability. In '1', the probability is calculated as the number of successful outcomes divided by the total possible number of outcomes. In this classical approach, the probability assignments are based on equally likely outcomes and the entire sample space is known from the start.

The situation in '2' is no longer as well determined as in '1'. It is necessary to obtain historical data for the salesman in question and estimate the required probability by dividing the number of successful sales by the total number of households visited. This frequency approach still relies on the existence of data and its applications are more realistic than those of the classical methodology.

For '3' it is very unclear that a probability can be assigned. Historical data is most likely unavailable or insufficient for the frequency approach. The statistician might have to revert to informed educated guesses. This is quite permissible and reflects the analyst's prior opinion. This approach lends itself to a Bayesian methodology.

One should note that the rules of probability theory remain the same regardless of the method used to estimate the probability of events.

## KINEMATICS

### Definitions

**Displacement** is the position of a point relative to a fixed origin O. It is a vector. The SI Unit is the metre (m). Other metric units are centimeter (cm), kilometer (km).

**Velocity** is the rate of change of displacement with respect to time. It is a **vector**. The SI Unit is **metre per second** ( $\text{ms}^{-1}$ ). Other metric units are  $\text{cms}^{-1}$ ,  $\text{kmh}^{-1}$ .

**Speed** is the magnitude of the velocity and is a scalar quantity.

**Uniform velocity** is the constant speed in a fixed direction.

**Average velocity** =  $\frac{\text{change in displacement}}{\text{time taken}}$

**Average speed** =  $\frac{\text{total distance travelled}}{\text{time taken}}$

**Acceleration** is the rate of change of velocity with respect to time. It is a **vector**. The SI Unit is **metre per second square** ( $\text{ms}^{-2}$ ). Other metric units are  $\text{cms}^{-2}$ ,  $\text{kmh}^{-2}$ .

**Negative acceleration** is also referred to as retardation.

**Uniform acceleration** is the constant acceleration in a fixed direction.

**Motion in one dimension** – When a particle moves in **one dimension**, that is, along a straight line, it has only two possible directions in which to move. Positive and negative signs are used to identify the two directions.

**Vertical motion under gravity** – this is a special case of uniform acceleration in a straight line. The body is thrown **vertically upward**, or falling **freely downward**. This uniform acceleration is due to **gravity** and acts vertically downwards towards the centre of the earth. It is denoted by **g** and may be approximated by  $9.8 \text{ ms}^{-2}$ .

## GRAPHS IN KINEMATICS

A **displacement-time** graph for a body moving in a straight line shows its displacement  $x$  from a fixed point on the line plotted against time,  $t$ . The **velocity**  $v$  of the body at time,  $t$  is given by the **gradient** of

the graph since  $\frac{dx}{dt} = v$ .

The **displacement-time** graph for a body moving with **constant velocity** is a **straight line**. The velocity,  $v$  of the body is given by the gradient of the line.

The **displacement-time** graph for a body moving with **variable velocity** is a **curve**.

The velocity at any time,  $t$  may be estimated from the gradient of the tangent to the curve at that time. The average velocity between two times may be estimated from the gradient of the chord joining them.

**Velocity-time graph** for a body moving in a straight line shows its velocity  $v$  plotted against time,  $t$ .

The **acceleration**,  $a$  of a body at time,  $t$  is given by the **gradient** of the graph at  $t$ , since  $a = \frac{dv}{dt}$ .

The **displacement** in a time interval is given by the **area** under the **velocity-time** graph for that time interval.

The **velocity-time** graph for a body moving with **uniform acceleration** is a **straight line**. The acceleration of the body is given by the gradient of the line.

## ◆ GUIDELINES FOR THE SCHOOL BASED ASSESSMENT

### RATIONALE

School Based Assessment is an integral part of student assessment in the course covered by this syllabus. It is intended to assist students in acquiring certain knowledge, skills and attitudes that are associated with the subject. The activities for the School Based Assessment are linked to the syllabus and should form part of the learning activities to enable the student to achieve the objectives of the syllabus.

During the course of study for the subject, students obtain marks for the competence they develop and demonstrate in undertaking their School Based Assessment assignments. These marks contribute to the final marks and grades that are awarded to students for their performance in the examination.

The guidelines provided in this syllabus for selecting appropriate tasks are intended to assist teachers and students in selecting assignments that are valid for the purpose of School Based Assessment. The guidelines provided for the assessment of these assignments are intended to assist teachers in awarding marks that are reliable estimates of the achievement of students in the School Based Assessment component of the course. In order to ensure that the scores awarded by teachers are in line with the CXC standards, the Council undertakes the moderation of a sample of the School Based Assessment assignments marked by each teacher.

School Based Assessment provides an opportunity to individualise a part of the curriculum to meet the needs of students. It facilitates feedback to the student at various stages of the experience. This helps to build the self-confidence of students as they proceed with their studies. School Based Assessment also facilitates the development of the critical skills and abilities that are emphasised by this CSEC subject and enhances the validity of the examination on which candidate performance is reported. School Based Assessment, therefore, makes a significant and unique contribution to the development of relevant skills and the testing and rewarding of students for the development of those skills.

The Caribbean Examinations Council seeks to ensure that the School Based Assessment scores are valid and reliable estimates of accomplishment. The guidelines provided in this syllabus are intended to assist in doing so.

### Assignment

The School Based Assessment consists of one project to be marked by the teacher in accordance with CXC guidelines.

There are two types of project.

**Project A** is based on applying mathematical concepts, skills and procedures from any topic (s) in order to understand, describe or explain a real world phenomenon. The project is theory based and no data collection is required.

**Project B** is based on applying mathematical concepts, skills and procedures from any topic (s) in order to understand, describe or explain a real world phenomenon. The project is experiment based and involves the collection of data.

Candidates should complete one project, either Project A or Project B.

### Role of the Teacher

The role of teacher is to:

- (i) Suggest the project for the School Based Assessment.
- (ii) provide guidance throughout the life of the projects. The teacher should work with candidates to develop a project management chart with definite time lines for achieving clearly identified objectives, from project inception to project completion.
- (iii) guide the candidate through the SBA by helping to clarify the problem or by discussing possible approaches to solving the problem. Teachers, while giving guidance, should guard against providing a complete solution to the problem for the candidate or prescribing a specific format that should be followed.
- (iv) ensure that the project is developed as a continuous exercise that occurs during scheduled class hours as well as outside class times.
- (v) at a time to be determined by the teacher the relevant component will be assessed and the marks recorded. Hardcopies of the completed documents should be kept by both the teacher and student. The teacher should use the mark scheme provided by CXC and include any comments pertinent to the conduct of the assessment.

## ◆ ASSESSMENT CRITERIA

Candidates should complete one project, either Project A or Project B.

### Project A

The following are the guidelines for assessing this project.

1. Each candidate pursuing Additional Mathematics can complete a project which will be based on applying the mathematical concepts, skills and procedures from any topic (s) contained in any of the sections or their combinations in order to understand, describe or explain a real world phenomenon.

The project will be presented in the form of a report and will have the following parts.

- (i) A statement of the problem

A real world problem in Mathematics chosen from any subject or discipline such as Science, Business or the Arts. The student must solve the problem using Specific Objectives completed in the course. This solution will involve either a proof or direct application of the concepts. (Data collection is not usually required for this project. Any necessary data should be given in the problem statement.)

- (ii) Identification of important elements of the problem.

- (iii) Mathematical Formulation of the problem.

- (iv) Selection of a method to solve the problem. This should involve use of Specific Objectives.

- (v) Solution of the resulting mathematical problem.

- (vi) Interpretation of the solution in related real world context.

- (vii) Conclusions reached.

2. The project will be graded out of a total of 20 marks.

- (i) Clarity of the title of the real world problem being studied.

- (ii) Scope/purpose of the problem.

- (iii) Mathematical formulation of the problem.

- (iv) The problem solution including explanation at each step.

- (v) Applications of the solution or proof to the given real world problem.

- (vi) Discussion of findings and conclusion statement (this should include suggestions for further analysis of the topic).
- (vii) Presentation (including an optional oral question and answer time with the teacher).

### Assessing Project A

The project will be graded out a total of 20 marks and marks will be allocated to each task as outlined below.

#### Project Descriptors

1. **Project Title**
  - Title is clear and concise, and relates to real world problem (1) [1]
2. **Purpose of Project/Problem Statement**
  - Purpose is clearly stated and is appropriate in level of difficulty (1) [1]
3. **Mathematical Formulation**
  - Identifies all the important elements of the problem and shows complete understanding of the relationships among elements (1) [4]
  - Shows complete understanding of the problem's mathematical concepts and principles (1)
  - Uses appropriate mathematical terminology and notations to model the problem mathematically (1)
  - Uses appropriate Mathematical model/methods chosen (1)
4. **The Problem Solution**
  - Assumptions are clearly stated (1) [7]
  - Proofs are well established (1)
  - Diagrams are appropriate and clearly labelled (1)
  - Explanations are sufficient and clearly expressed (1)
  - Theorems are appropriate and Formulae are relevant to the solution (1)
  - Calculations are precise without errors (1)
  - Solution is clearly stated (1)
5. **Application of Solution**
  - Applies the solution or proof to the given real world problem (1) [2]
  - Shows that solution of proof to given problem is valid (1)



6. **Discussion of Findings/Conclusion**

- Discussion is worthwhile (1)
- Conclusion is valid and useful (1) [3]
- Suggestions for future analysis in related areas are incorporated. (1)

7. **Overall Presentation**

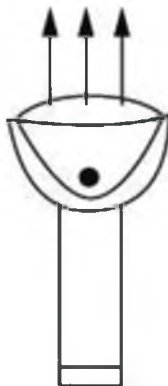
- Presentation is clear and communicates information in a logical way using correct grammar, mathematical jargon and symbols. (2) [2]
- Communicates information in a logical way using correct grammar, mathematical jargon and symbols some of the time. (1)

Total 20 marks

**PROJECT A - EXEMPLAR**

**Title: Use of a Parabolic Reflector in Making a Torch Light.**

**Problem:** A physicist wants to make an instrument that will reflect rays of light from a minute light source in a **vertical beam parallel** to the axis of symmetry as shown in Figure 1.



**Figure 1: Physicist's Instrument**

Two crucial decisions he must make concern:

- (i) The shape of the reflector to be used.
- (ii) The position of the light source.

A. Explain why the physicist should use a parabolic reflector with equation  $y = \frac{x^2}{4p}$  and position his light source at the Focus  $(0, p)$ .

- B. If the physicist uses the parabolic reflector, as shown in Figure 2, how far from the vertex should the light source be placed on the y axis to produce a beam of parallel rays?

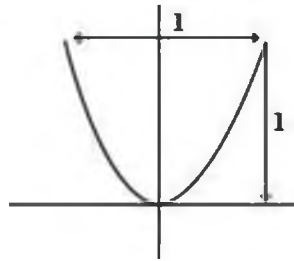


Figure 2: Parabolic Reflector

(Assume that when light is reflected from a point  $P$ , the angle  $\alpha$  between an incoming ray and the tangent line at  $P$  equals the angle  $\beta$  between the outgoing ray and the tangent line at  $P$ .)

**Mathematical Formulation:**

- A 1. Show that if we have the distance  $FP$  is equal to  $PD$  then the equation of the curve

is  $y = \frac{x^2}{4p}$  (see Figure 3).

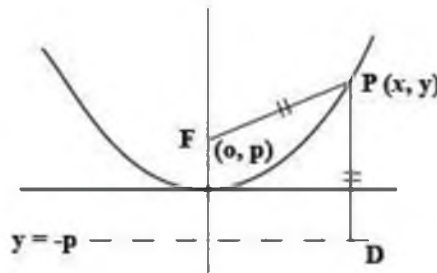


Figure 3

2. Show that the tangent line at  $P_0$  intersects the y axis at  $Q(0, -y_0)$

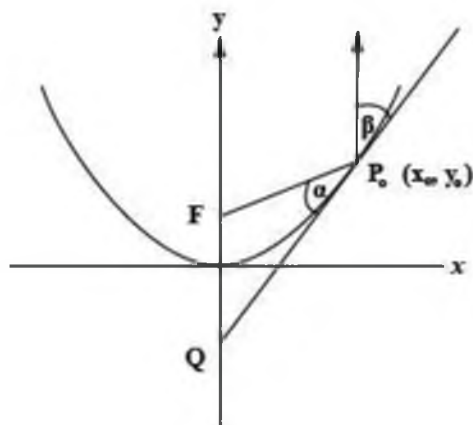
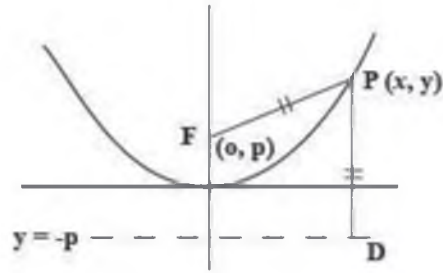


Figure 4

3. Prove that the triangle whose vertices are  $Q, F, P_0$  is isosceles.
4. Show that  $\alpha = \beta$  in Figure 4.
5. Deduce that the physicist should use a parabolic reflector with equation  $y = \frac{x^2}{4p}$  and position his light source at the Focus  $(0, p)$ . for his instrument to work.
6. Apply to the specific example given in Figure 2.

#### Solution of Problem

- A. To show that the physicist should use a parabolic reflector with equation  $y = \frac{x^2}{4p}$  and position his light source at the focus,  $F(0, p)$ .
1. Show that if for any point  $P(x, y)$  on the curve, the distance  $FP$  is equal to  $PD$ , then the equation of the curve is  $y = \frac{x^2}{4p}$  where  $F$  is the point  $(0, p)$ .



Show that  $y = \frac{x^2}{4p}$  if  $FP = PD$

**PROOF**

$$FP = \sqrt{(x - 0)^2 + (y - p)^2}$$

$$PD = \sqrt{(y - (-p))^2}$$

$$\therefore \sqrt{x^2 + (y - p)^2} = \sqrt{(y + p)^2}$$

$$\therefore x^2 + (y - p)^2 = (y + p)^2 \text{ (squaring both sides)}$$

$$\therefore x^2 + y^2 + p^2 - 2yp = y^2 + p^2 + 2yp$$

$$\therefore x^2 = 4yp, \therefore y = \frac{x^2}{4p}$$

The tangent line to the curve,  $y = \frac{x^2}{4p}$ , at the point  $P_0$  is shown in Figure 4. Show that the tangent line at  $P_0$  intersects the  $y$ -axis at  $Q(0, -y_0)$ .

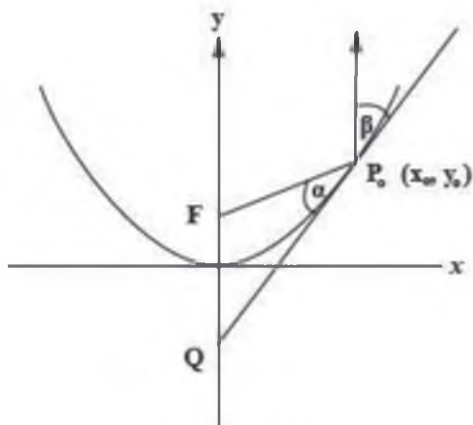


Figure 4

The tangent line at  $P_0$  intersects the y-axis at  $Q(0, -y_0)$

**PROOF**

We find the equation of the tangent at  $P_0$ :

$$\frac{dy}{dx} = \frac{2x}{4p} = \frac{x}{2p}$$

$$\left. \frac{dy}{dx} \right|_{x=x_0} = \frac{x_0}{2p} = \text{slope of tangent.}$$

Since  $(x_0, y_0)$  is a point on the parabola  $y_0 = \frac{x_0^2}{4p}$

The equation of the tangent line through  $(x_0, \frac{x_0^2}{4p})$  is therefore:

$$y - \frac{x_0^2}{4p} = \frac{x_0}{2p}(x - x_0) \quad (i)$$

To find where this line intersects the y-axis we substitute  $x = 0$  into (i)

$$\therefore y - \frac{x_0^2}{4p} = \frac{x_0}{2p}(-x_0)$$

$$\therefore y : \frac{x_0^2}{4p} - \frac{x_0^2}{2p} = \frac{-x_0^2}{4p} = -y_0$$

$\therefore$  The co-ordinates of Q is  $(0, -y_0)$

2. Show that the triangle whose vertices are Q, F,  $P_0$  is isosceles.

To show that  $\Delta Q, F, P_0$  is an isosceles triangle, we show that  $FQ = FP_0$

**PROOF**

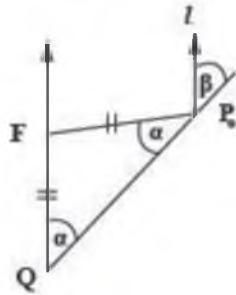
$$FQ = \sqrt{(p - -y)^2} = \sqrt{(p + y_0)^2} = p + y_0$$

$$FP_0 = \sqrt{(x_0 - 0)^2 + (y_0 - p)^2} = \sqrt{x_0^2 + (y_0 - p)^2}$$

$$\begin{aligned} FP_0 &= \sqrt{4py_0 + (y_0 - p)^2} \\ &= \sqrt{4py_0 + y_0^2 - 2py_0 + p^2} \\ &= \sqrt{y_0^2 + 2py_0 + p^2} = \sqrt{(y_0 + p)^2} = y_0 + p \end{aligned}$$

$\therefore FQ = FP_0$  and  $\Delta QFP_0$  is isosceles

3. Show that that  $\alpha = \beta$ .



**Figure 5**

Since the base angles of an isosceles  $\Delta$  are equal  $\angle FQP_0 = \angle FP_0Q = \alpha$ .

But  $\alpha$  and  $\beta$  are corresponding angles since the line through  $P_0$  is parallel to the y-axis, therefore,  $\alpha = \beta$ .

4. Show that the physicist should use a parabolic reflector with equation  $y = \frac{x^2}{4p}$  and position his light source at the focus,  $F(0, p)$ .

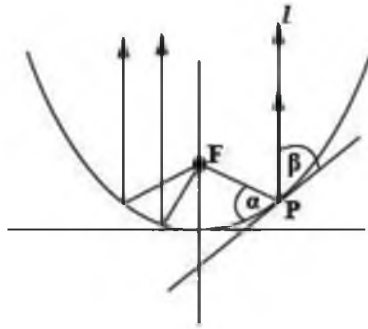


Figure 6

The physicist knows that for any ray from  $F$  striking the reflector at  $P$ ,  $\alpha = \beta$  (assumption: angle of incidence equals angle of reflection).

But from part earlier we know that for the parabola when  $\alpha = \beta$ , the line  $l$  will be parallel to the axis of symmetry if  $F$  is  $(0, p)$ .

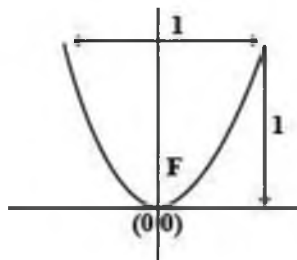
Therefore all rays from the minute light source at  $F$  that are incident on the parabola will be reflected in a direction parallel to the  $y$  axis.

#### Solution to Part A

Thus the physicist should use a parabolic reflector  $y = \frac{x^2}{4p}$  with light source at focus,  $F$ , to produce a beam of parallel rays.

#### **Application of the Solution**

- B. How far from the vertex should the light source be placed on the  $y$ -axis to produce a beam of parallel rays?



For the given parabola  $y = 1$  when  $x = 1/2$

$$\text{Since } y = \frac{x^2}{4p},$$

$$1 = \frac{\left(\frac{1}{2}\right)^2}{4p}$$

$$4p = 1/4,$$

$$p = \frac{1}{16}$$

The light source should be placed  $\frac{1}{16}$  units from the vertex in order to produce a beam of parallel rays.

### Discussion of Findings

To make an instrument that would reflect a minute light source in a vertical beam parallel to the axis of the symmetry, we have shown that if the physicist uses a parabolic reflector with equation  $y = \frac{x^2}{4p}$  and positions his light source at the focus,  $F(0, p)$  then his instrument will work and rays of light will be reflected in a vertical beam.

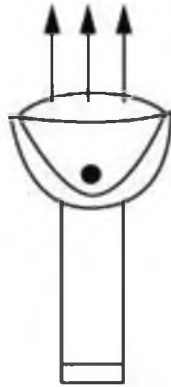


Figure 1

For example if parabolic reflection has the dimensions given in Figure 2, he should position his light source at  $\frac{1}{16}$  units from the vertex of the parabola. This is useful in making torch lights.

### Conclusion and Suggestions

By understanding the optical properties of a parabola the physicist will know where to position his light source.

It would also be interesting to investigate the optical properties of other curves such as the ellipse, circle and hyperbola to see how their properties can be used to help construct instrument such as reflecting headlights, telescopes and microscopes.



## **Project B**

The following are guidelines for assessing this project.

1. Each candidate pursuing Additional Mathematics can complete a project which will be based on applying the mathematical concepts, skills and procedures from any topic(s) in order to understand, describe or explain a real world phenomenon. This project is experiment based and involves the collection of data.

The project will be presented in the form of a report and will have the following parts:

- (i) A statement of the problem. A real world problem in Mathematics chosen from any subject or discipline such as Science, Business or the Arts. The student must solve the problem using Specific Objectives completed in the course. This solution will involve data collection required for this project.
  - (ii) Identification of important elements of the problem.
  - (iii) Formulation of a systematic strategy for representing the problem
  - (iv) Data collection appropriate for solving the problem.
  - (v) An analysis of the data, information and measurements.
  - (vi) Solution of the resulting mathematical problem.
  - (vii) Conclusions reached.
2. The project will be graded out of a total of 20 marks:
    - (i) Clarity of the title of the real world problem being studied.
    - (ii) Scope/purpose of the problem.
    - (iii) Method of data collection.
    - (iv) Presentation of data.
    - (v) Mathematical knowledge/analysis of data.
    - (vi) Discussion of findings/conclusions.
    - (vii) Presentation.

## ASSESSING PROJECT B

The project will be graded out a total of 20 marks and marks will be allocated to each task as outlined below.

### Project Descriptors

1. **Project Title** [1 ]
  - Titled is clear and concise, and relates to real world problem (1)
2. **Purpose of Project** [2 ]
  - Purpose is clearly stated and is appropriate in level of difficulty (1)
  - Appropriate variables identified (1)
3. **Method of Data Collection** [2 ]
  - Data collection method clearly described (1)
  - Data collection method is appropriate and without flaws (1)
4. **Presentation of Data** [3]
  - At least one table and one graph/chart used (1)
  - Data clearly written, labelled, unambiguous and systematic (1)
  - Graphs, figures, tables and statistical/mathematical symbols used appropriately (1)
5. **Mathematical Knowledge/Analysis of Data** [5]
  - Appropriate use of mathematical concepts demonstrated (1)
  - Accurate use of mathematical concepts demonstrated (1)
  - Some analysis attempted (1)
  - Analysis is coherent (1)
  - Analysis used a variety (two or more) of approaches (1)
6. **Discussion of Findings/Conclusion** [5]
  - Statement of most findings are clearly identified (1)
  - Statement follows from data gathered/solution of problem (1)
  - Conclusion based on findings and related to purposes of project (1)
  - Conclusion is valid (1)
  - Suggestions for future analysis in related areas (1)
7. **Overall Presentation** [2]
  - Communicates information in a logical way using correct grammar, (2)

- mathematical jargon and symbols most of the time
- Communicates information in a logical way using correct grammar, (1) mathematical jargon and symbols some of the time

Total 20 marks

## **PROJECT B – EXEMPLAR**

### **Title**

#### **Simple experiments to determine the fairness of an ordinary game die.**

#### Statement of Task

Classical probability states that the probability of any of the 6 faces of an ordinary cubical game die landing with a distinct face uppermost after being thrown is  $\frac{1}{6}$ . It is not unusual for one throwing an ordinary gaming die to observe that one particular face lands uppermost with more frequency than any of the other faces.

Is this sufficient reason for one to conclude that the die may be biased? It may be by chance that this phenomenon occurs, or, perhaps the manner in which the die is thrown has an effect on its outcome. An experiment of this nature may be affected by factors that vary because of the non-uniform manner in which it is conducted.

This project aims to carry out some simple experiments to determine whether or not some varying factors of the manner in throwing the die do in fact influence the outcomes.

#### Data Collection

An ordinary 6-face gaming die was chosen for this experiment. 120 throws were made for each experiment, using each of the following methods:

- (i) holding the die in the palm of the hand and shaking it around a few times before throwing it onto a varnished table top;
- (ii) placing the die in a Styrofoam drinking cup, shaking it around a few times before throwing it onto a varnished table top;
- (iii) placing the die in a smooth metal drinking cup, shaking it around a few times before throwing it onto a varnished table top;
- (iv) holding the die in the palm of the hand and shaking it around a few times before throwing it onto a linen covered table top;
- (v) placing the die in a Styrofoam drinking cup, shaking it around a few times before throwing it onto a linen covered table top;

- (vi) placing the die in a smooth metal drinking cup, shaking it around a few times before throwing it onto a linen covered table top;

After each experiment the frequencies of the numbers landing uppermost were recorded in tabular form.

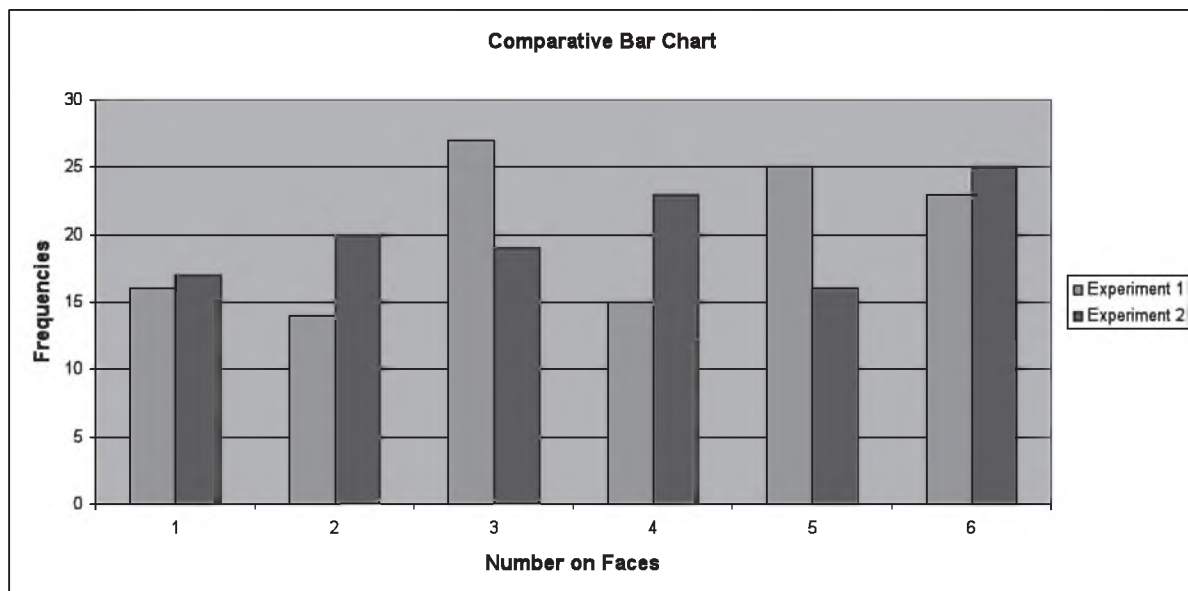
In each of these experiments the number of times the die was shaken before throwing was not predetermined, nor was any other deliberate consideration applied in the subsequent throws. Every effort was taken to avoid bias in each of the experiments.

The following table shows the results of the experiments carried out.

# on face	1	2	3	4	5	6
Frequencies – Exp (i)	16	14	27	15	25	23
Frequencies – Exp (ii)	17	20	19	23	16	25
Frequencies – Exp (iii)	18	25	20	19	25	13
Frequencies – Exp (iv)	16	21	20	29	13	21
Frequencies – Exp (v)	13	20	27	18	19	23
Frequencies – Exp (vi)	14	24	17	24	25	16
Total frequencies	94	124	130	128	123	121

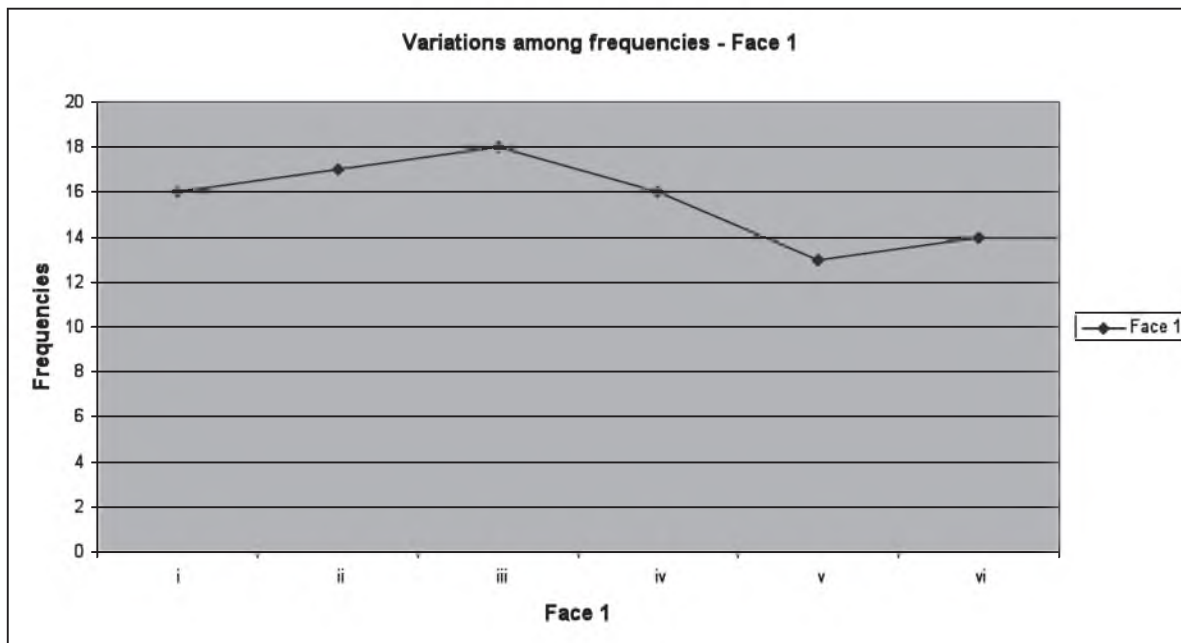
#### Presentation of Data

The following comparative bar chart illustrates the variations of frequencies for obtaining the numbers 1 through 6 on the uppermost face for experiments (i) and (ii).



Graphs to illustrate experiments (iii), (iv), (v) and (vi) may be shown as well.

The following line graph illustrates the variations among the frequencies for face 1.



Graphs for each of faces 2, 3, 4, 5, and 6 may be shown.

#### Mathematical Knowledge/Analysis of Data

Choosing to use the different methods for carrying out these experiments, as described in Data Collection, took into account that different conditions of the throws of the die may have significant influences in the outcomes of these throws. The size of the cups chosen may have a particular influence on these outcomes. The inside surface of the two types of cups chosen are also factors that may influence these outcomes. The number of times the die is tossed around in the palm of the hand and/or the number of times it is tossed around in the cups may influence these outcomes. The different coverings of the surface of the table top may also influence these outcomes.

In the absence of more in-depth and elaborate statistical techniques, these simple experiments were intended to give some idea of the theory of classical probability. The limiting relative frequency of an event over a long series of trials is the conceptual foundation of the frequency interpretation of probability. In this framework, it is assumed that as the length of the series increases without bound, the fraction of the experiments in which we observe the event will stabilize.

120 throws under each of the conditions selected should allow for simple comparison of the observed and theoretical frequencies.

Using the principle of relative probability, the following table shows the probability distribution for Experiment (i) and the theoretical probability of obtaining any of the faces numbered 1, 2, 3, 4, 5, 6 landing uppermost.

# on face	1	2	3	4	5	6
Relative probability	$\frac{2}{15} = 0.13$	$\frac{7}{60} = 0.12$	$\frac{9}{40} = 0.23$	$\frac{1}{8} = 0.13$	$\frac{5}{24} = 0.21$	$\frac{23}{120} = 0.19$
Theoretical probability	$\frac{1}{6} = 0.17$	$\frac{1}{6} = 0.17$	$\frac{1}{6} = 0.17$	$\frac{1}{6} = 0.17$	$\frac{1}{6} = 0.17$	$\frac{1}{6} = 0.17$

Comparisons of the differences of the observed and theoretical frequencies for 120 throws of the die under the conditions described should be considered as sufficient for an explanation of any significant variation in determining whether the die was biased in favour of any particular face. Barring any significant variation among the relative frequencies, it may be reasoned that the die is not biased.

The relative probabilities can also be calculated for Experiments (ii) through (vi)

Furthermore we can combine the results of all six experiments to arrive at an overall probability for each face as shown in the table below:

# on face	1	2	3	4	5	6
Relative frequency	$\frac{94}{720} = 0.13$	$\frac{124}{720} = 0.17$	$\frac{130}{720} = 0.18$	$\frac{128}{720} = 0.18$	$\frac{123}{720} = 0.17$	$\frac{121}{720} = 0.17$

The above table clearly shows that the relative frequency of each face is close to the true probability (0.17) when the number of trials (720) is large. This is strong evidence to claim that the die is unbiased even though there were differences among the observed frequencies for the six experiments.

Further analysis must be taken in light of any limitations that the project may have. Considering the mean and standard deviation of each of these experiments, account may be taken of the size of the variations of the observed and theoretical values. This aspect may explain any significant variation from the expected mean and variance of these outcomes

The standard deviations for the frequencies of faces 1 through 6 for Experiments (i), (ii), (iii), (iv), (v) and (vi) are 1.67, 1.71, 1.62, 1.63, 1.63 and 1.60 respectively.

Except for Face #2 and to a lesser extent (Face #1), the variances among the outcomes do not appear to suggest significant differences in the results.

### Conclusions

These experiments can be considered simplistic but reasonably effective for the purpose of determining bias in an ordinary gaming die. The number of throws, 120, may be considered sufficient for obtaining relative frequencies and relative probability for the experiments. Increasing the number of throws should result in observed frequencies very close to the theoretical frequencies.

Further statistical analyses can explain variations between the observed and theoretical results. These experiments may be refined by using other methods of throwing the die. Results can be compared for similarity among these results and for a reasonable conclusion about fairness of the die.

### Procedures for Reporting and Submitting School Based Assessment

- (i) Teachers are required to record the mark awarded to each candidate under the appropriate profile dimension on the mark sheet provided by CXC. The completed mark sheets should be submitted to CXC no later than April 30 of the year of the examination.

**Note: The school is advised to keep a copy of the project for each candidate as well as copies of the mark sheets.**

- (ii) Teachers will be required to submit to CXC copies of the projects of a sample of candidates as indicated by CXC. This sample will be re-marked by CXC for moderation purposes.

### Moderation of School Based Assessment

The candidate's performance on the project will be moderated. The standard and range of marks awarded by the teacher will be adjusted where appropriate. However, the rank order assigned by the teacher will be adjusted only in special circumstances and then only after consideration of the data provided by the sample of marked projects submitted by the teacher and re-marked by CXC.

## ◆ RESOURCES

The following is a list of books and other printed material that might be used for Additional Mathematics. The list is by no means exhaustive. Each student should have access to at least one text.

Talbert, J. F. And Heng, H. H.

*Additional Mathematics – Pure and Applied*, Singapore:  
Longman Publishers, 1991.

Website:

[http://www.saskschools.ca/curr\\_content/physics30kindy n/ for kinematics](http://www.saskschools.ca/curr_content/physics30kindy n/ for kinematics)

## ◆ GLOSSARY

### KEY TO ABBREVIATIONS

K - Knowledge  
C - Comprehension  
R - Reasoning

WORD	DEFINITION	NOTES
analyse	examine in detail	
annotate	add a brief note to a label	Simple phrase or a few words only.
apply	use knowledge/principles to solve problems	Make inferences/conclusions.
assess	present reasons for the importance of particular structures, relationships or processes	Compare the advantages and disadvantages or the merits and demerits of a particular structure, relationship or process.
calculate	arrive at the solution to a numerical problem	Steps should be shown; units must be included.
classify	divide into groups according to observable characteristics	
comment	state opinion or view with supporting reasons	
compare	state similarities and differences	An explanation of the significance of each similarity and difference stated may be required for comparisons which are other than structural.
construct	use a specific format to make and/or draw a graph, histogram, pie chart or other representation using data or material provided or drawn from practical investigations, build (for example, a model), draw scale diagram	Such representations should normally bear a title, appropriate headings and legend.



WORD	DEFINITION	NOTES
deduce	make a logical connection between two or more pieces of information; use data to arrive at a conclusion	
define	state concisely the meaning of a word or term	This should include the defining equation/formula where relevant.
demonstrate	show; direct attention to...	
derive	to deduce, determine or extract from data by a set of logical steps some relationship, formula or result	This relationship may be general or specific.
describe	provide detailed factual information of the appearance or arrangement of a specific structure or a sequence of a specific process	Description may be in words, drawings or diagrams or any appropriate combination. Drawings or diagrams should be annotated to show appropriate detail where necessary.
determine	find the value of a physical quantity	
design	plan and present with appropriate practical detail	Where hypotheses are stated or when tests are to be conducted, possible outcomes should be clearly stated and/or the way in which data will be analyzed and presented.
develop	expand or elaborate an idea or argument with supporting reasons	
diagram	simplified representation showing the relationship between components	
differentiate/distinguish (between/among)	state or explain briefly those differences between or among items which can be used to define the items or place them into separate categories	
discuss	present reasoned argument; consider points both for and against; explain the	

WORD	DEFINITION	NOTES
	relative merits of a case	
draw	make a line representation from specimens or apparatus which shows an accurate relation between the parts	In the case of drawings from specimens, the magnification must always be stated.
estimate	make an approximate quantitative judgement	
evaluate	weigh evidence and make judgements based on given criteria	The use of logical supporting reasons for a particular point of view is more important than the view held; usually both sides of an argument should be considered.
explain	give reasons based on recall; account for	
find	locate a feature or obtain as from a graph	
formulate	devise a hypothesis	
identify	name or point out specific components or features	
illustrate	show clearly by using appropriate examples or diagrams, sketches	
interpret	explain the meaning of	
investigate	use simple systematic procedures to observe, record data and draw logical conclusions	
justify	explain the correctness of	
label	add names to identify structures or parts indicated by pointers	
list	itemize without detail	
measure	take accurate quantitative readings using appropriate instruments	

<b>WORD</b>	<b>DEFINITION</b>	<b>NOTES</b>
name	give only the name of	No additional information is required.
note	write down observations	
observe	pay attention to details which characterize a specimen, reaction or change taking place; to examine and note scientifically	Observations may involve all the senses and/or extensions of them but would normally exclude the sense of taste.
outline	give basic steps only	
plan	prepare to conduct an investigation	
predict	use information provided to arrive at a likely conclusion or suggest a possible outcome	
record	write an accurate description of the full range of observations made during a given procedure	This includes the values for any variable being investigated; where appropriate, recorded data may be depicted in graphs, histograms or tables.
relate	show connections between; explain how one set of facts or data depend on others or are determined by them	
sketch	make a simple freehand diagram showing relevant proportions and any important details	
state	provide factual information in concise terms outlining explanations	
suggest	offer an explanation deduced from information provided or previous knowledge. (... a hypothesis; provide a generalization which offers a likely explanation for a set of data or observations.)	No correct or incorrect solution is presumed but suggestions must be acceptable within the limits of scientific knowledge.
use	apply knowledge/principles to solve problems	Make inferences/conclusions.

**Western Zone Office**  
**3 May 2010**





# CARIBBEAN EXAMINATIONS COUNCIL

Caribbean Advanced Proficiency Examination  
CAPE®



## ADDITIONAL MATHEMATICS

### Specimen Papers and Mark Schemes/Keys

**Specimen Papers:**

- Unit 1, Paper 01  
(Mark Scheme included)  
Unit 1, Paper 02  
Unit 1, Paper 03/2

**Mark Schemes and Keys:**

- Unit 1, Paper 02  
Unit 1, Paper 03/2



**CARIBBEAN EXAMINATIONS COUNCIL**

**SECONDARY EDUCATION CERTIFICATE  
EXAMINATION**

**ADDITIONAL MATHEMATICS**

**SPECIMEN PAPER**

**Paper 01 – General Proficiency**

*90 minutes*

**READ THE FOLLOWING INSTRUCTIONS CAREFULLY.**

1. This paper consists of **45** items. You will have 90 minutes to answer them.
2. In addition to this test booklet, you should have an answer sheet.
3. Each item in this test has four suggested answers, lettered (A), (B), (C) and (D). Read each item you are about to answer and decide which choice is best.
4. On your answer sheet, find the number which corresponds to your item and shade the space having the same letter as the answer you have chosen. Look at the sample item below.

Sample Item

Evaluate  $(4^{-2})^2 \div (\frac{1}{16})^2$

- (A)  $4^{-2}$
- (B)  $4^{-1}$
- (C)  $4^0$
- (D)  $4^2$

Sample Answer

A B ● D

The best answer to this item is “ ”, so answer space (C) has been shaded.

5. If you want to change your answer, erase it completely before you fill in your new choice.
6. When you are told to begin, turn the page and work as quickly and as carefully as you can. If you cannot answer an item, omit it and go on to the one. You can return later to the item omitted. Your score will be the number of correct answers produced.
7. You may do any rough work in the booklet.
8. You may use a silent non-programmable calculator to answer questions.

1. Given that  $f(x) = x^3 + 2x^2 - 5x + k$ , and that  $x - 2$  is a factor of  $f(x)$  then  $k$  is equal to

- (A) -6
- (B) -2
- (C) 2
- (D) 6

2.  $a(b + c) - b(a + c)$  is equal to

- (A)  $a(c - b)$
- (B)  $a(b - c)$
- (C)  $c(a - b)$
- (D)  $c(b - a)$

3. The value of  $\sum_{r=1}^{20} (3r - 1)$  is

- (A) 590
- (B) 610
- (C) 650
- (D) 1220

4. A teacher illustrates AP's by cutting a length of string into 10 pieces so that the lengths of the pieces are in arithmetic progression and the entire length of the string is used up exactly. If the first piece measures 30 cm and the fourth piece measures 24 cm, the total length of the string is

- (A) 60 cm
- (B) 210 cm
- (C) 240 cm
- (D) 390 cm

5. The first term of a GP is 16 and the fifth term is 81. Given that the common ratio is positive, the value of the 4<sup>th</sup> term is

- (A)  $\frac{81}{16}$
- (B) 24
- (C) 54
- (D) 64

6. The first four terms of a convergent GP is given by 81, 27, 9, 3. The sum to infinity of this GP is

- (A) 54
- (B) 120.5
- (C) 121.5
- (D) 243

7. Given that  $2 \times 4^{x+1} = 16^{2x}$ , the value of  $x$  is

- (A) -1
- (B)  $\frac{1}{4}$
- (C)  $\frac{1}{3}$
- (D)  $\frac{1}{2}$

8.  $\sqrt[n]{2 \times 4^m}$  is equal to

- (A)  $\sqrt[n]{8^m}$
- (B)  $2^{n+2m}$
- (C)  $2^{n+mn}$
- (D)  $2^{\frac{2m+1}{n}}$

9. Given that  $\log_2 x + \log_2 (6x + 1) = 1$ , the value of  $x$  is

- (A)  $-\frac{2}{3}$
- (B)
- (C)  $\frac{2}{3}$
- (D)  $\frac{3}{2}$

10. The value of  $\log_4(8) - \log_4(2) + \log_4\left(\frac{1}{16}\right)$  is
- (A) -1  
(B)  $\frac{1}{2}$   
(C) 3  
(D) 4
11. The expression  $\frac{1 + \sqrt{3}}{\sqrt{3} - 1}$  when simplified is equal to
- (A) -1  
(B) 1  
(C)  $\frac{\sqrt{3} + 2}{2}$   
(D)  $\sqrt{3} + 2$
12.  $f(x) = -5 - 8x - 2x^2$ . By completing the square  $f(x)$  can be expressed as
- (A)  $2(x + 2)^2 - 4$   
(B)  $4 - 2(x - 2)^2$   
(C)  $3 - 2(x + 2)^2$   
(D)  $3 - 2(x - 2)^2$
13. The roots of the equation  $2x^2 - x + 1 = 0$  are
- (A) real and equal  
(B) real and distinct  
(C) not real and equal  
(D) not real and distinct
14. For  $-2 \leq x \leq 2$ , the maximum value of  $4 - (x + 1)^2$ , and the value of  $x$  for which  $4 - (x + 1)^2$  is maximum are respectively
- (A) 5 and 1  
(B) 2 and -1  
(C) 4 and -1  
(D) 4 and 1
15.  $f(x) = x(x + 5) + 6$ . Given that  $f(x)$  is one-to-one for  $x \geq k$ , the value of  $k$  is
- (A)  $-\frac{5}{2}$   
(B)  $-\frac{2}{5}$   
(C)  $\frac{2}{5}$   
(D)  $\frac{5}{2}$
16. If a function  $f$  is defined by  $f : x \rightarrow \frac{x + 3}{x - 1}$ ,  $x \neq 1$ , then  $f^{-1}(-4)$  is equal to
- (A) -1  
(B)  $\frac{1}{5}$   
(C) 1  
(D) 5
17. A function  $g$  is defined by  $g : x \rightarrow 3x - 1$ . Expressed in terms of  $a$ ,  $g(3a - 1)$  is
- (A)  $9a - 1$   
(B)  $3a - 4$   
(C)  $9a - 2$   
(D)  $9a - 4$



18. Functions  $f$  and  $g$  are defined by  
 $f : x \rightarrow 3x - 2$  and  
 $g : x \rightarrow \frac{12}{x} - 4, x \neq 0$ .
- The composite function  $fg$  is defined by
- (A)  $fg : x \rightarrow \frac{36}{x} - 4, x \neq 0$
- (B)  $fg : x \rightarrow \frac{12}{x} - 12, x \neq 0$
- (C)  $fg : x \rightarrow \frac{12}{x} - 6, x \neq 0$
- (D)  $fg : x \rightarrow \frac{36}{x} - 14, x \neq 0$
19. The range of values for which  
 $2x^2 < 5x + 3$  is
- (A)  $-\frac{1}{2} < x < 3$
- (B)  $\frac{1}{2} < x < 3$
- (C)  $x < -\frac{1}{2}$  and  $x < 3$
- (D)  $x > -\frac{1}{2}$  and  $x > 3$
20. The values of  $x$  which satisfy the  
inequality  $\frac{2x - 3}{x + 1} > 0$  are
- (A)  $x > -1$  and  $x > \frac{3}{2}$
- (B)  $x > \frac{3}{2}$
- (C)  $x < -1$  or  $x > \frac{3}{2}$
- (D)  $x > -1$
21. The coordinates of the points  $A$  and  $B$  are  
 $(2, -3)$  and  $(-10, -5)$  respectively. The  
perpendicular bisector to the line  $AB$  is  
given by the equation
- (A)  $x - 6y + 20 = 0$
- (B)  $6x + y + 28 = 0$
- (C)  $x + 6y - 20 = 0$
- (D)  $6x + y - 28 = 0$
22. The lines  $2y - 3x - 13 = 0$  and  
 $y + x + 1 = 0$  intersect at the point  $P$ ,  
where the coordinates of  $P$  are
- (A)  $(3, 2)$
- (B)  $(3, -2)$
- (C)  $(-3, -2)$
- (D)  $(-3, 2)$
23. The radius,  $r$ , and the coordinates of the  
centre,  $C$ , of the circle with equation  
 $x^2 + y^2 - 6x + 4y - 12 = 0$  are
- (A)  $r = 5, C(-2, 3)$ ,
- (B)  $r = 25, C(2, -3)$ ,
- (C)  $r = 12, C(-3, 2)$ ,
- (D)  $r = 5, C(3, -2)$ ,
24. If the length of the vector  $\mathbf{p} = 2\mathbf{i} - k\mathbf{j}$  is  
 $\sqrt{13}$  and  $k$  is real, then
- I.  $k = 3$
- II.  $k = -3$
- III.  $k = \sqrt{17}$
- IV.  $k = -\sqrt{17}$
- (A) I or II only
- (B) I or III only
- (C) I or IV only
- (D) II or IV only

25. The value of the real number  $t$  for which the two vectors  $\mathbf{a} = 4\mathbf{i} + t\mathbf{j}$  and  $\mathbf{b} = 2\mathbf{i} - 3\mathbf{j}$  are parallel is

(A)  $-6$

(B)  $-\frac{3}{4}$

(C)  $\frac{4}{3}$

(D)  $6$

26. The position vectors of  $A$  and  $B$  relative to an origin  $O$  are  $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$  and  $\begin{pmatrix} 7 \\ 4 \end{pmatrix}$  respectively. The acute angle  $AOB$  is given by

(A)  $\cos^{-1}\left(\frac{2}{\sqrt{65}}\right)$

(B)  $\cos^{-1}\left(\frac{\sqrt{26}}{13 \times 65}\right)$

(C)  $\cos^{-1}\left(\frac{\sqrt{2}}{\sqrt{65}}\right)$

(D)  $\cos^{-1}\left(\frac{26}{\sqrt{13} \sqrt{65}}\right)$

27. The trigonometrical expression  $\frac{1 + \sin x}{\cos x} + \frac{\cos x}{1 + \sin x}$  is identical to

(A)  $1$

(B)  $\frac{2}{\cos x}$

(C)  $\frac{1 + \sin x + \cos x}{\cos x(1 + \sin x)}$

(D)  $\frac{2}{\cos x(1 + \sin x)}$

28.  $\cos(A - B) - \cos(A + B) \equiv$

(A)  $2 \sin A \sin B$

(B)  $-2 \sin A \cos B$

(C)  $2 \cos A \sin B$

(D)  $2 \cos A \cos B$

29. If  $\sin \theta = \frac{15}{17}$  and  $\theta$  is obtuse, then  $\cos \theta$  is equal to

(A)  $-\frac{8}{15}$

(B)  $-\frac{8}{17}$

(C)  $\frac{8}{15}$

(D)  $\frac{8}{17}$

30. The smallest positive angle for which the equation  $\sin \theta + \cos \theta = 0$  is

(A)  $\frac{\pi}{4}$

(B)  $\frac{3\pi}{4}$

(C)  $\frac{5\pi}{4}$

(D)  $\frac{7\pi}{4}$

31. For  $0 \leq \theta \leq 2\pi$ , solutions for the equation  $4 \sin^2 \theta - 1 = 0$  exist in quadrants

(A)  $1, 2$  and  $3$

(B)  $1, 3$  and  $4$

(C)  $2, 3$  and  $4$

(D)  $1, 2, 3$  and  $4$

32.  $2 \sin\left(x - \frac{\pi}{2}\right)$  is equal to
- (A)  $2 \sin x - 2$   
 (B)  $-2 \cos x$   
 (C)  $2 \cos\left(x + \frac{\pi}{2}\right)$   
 (D)  $2 \sin x - \pi$
33. For which of the following ranges of values is  $f(x) = 2 + \cos 3x$  valid?
- (A)  $1 \leq f(x) \leq 3$   
 (B)  $-1 \leq f(x) \leq 1$   
 (C)  $-2 \leq f(x) \leq 2$   
 (D)  $0 \leq f(x) \leq 2$
34. For  $0 \leq x \leq 2\pi$ , the values of  $x$  which satisfy the equation  $2 \cos^2 x + 3 \sin x = 0$  are
- (A)  $x = \frac{\pi}{6}, x = \frac{5\pi}{6}$   
 (B)  $x = -\frac{\pi}{6}, x = -\frac{5\pi}{6}$   
 (C)  $x = \frac{7\pi}{6}, x = \frac{11\pi}{6}$   
 (D)  $x = \frac{5\pi}{6}, x = \frac{7\pi}{6}$
35. Given that  $y = (3x - 2)^3$ , then  $\frac{dy}{dx} =$
- (A)  $3(3x - 2)^2$   
 (B)  $3(3x)^2$   
 (C)  $3(3x - 2)^3$   
 (D)  $9(3x - 2)^2$
36. Given that  $y = \frac{3x + 5}{2x - 11}$ , then  $\frac{dy}{dx} =$
- (A)  $\frac{(3x + 5)(2) + (2x - 11)(3)}{(2x - 11)^2}$   
 (B)  $\frac{(2x - 11)(3) + (3x + 5)(2)}{(2x - 11)^2}$   
 (C)  $\frac{(2x - 11)(3) - (3x + 5)(2)}{(2x - 11)^2}$   
 (D)  $\frac{(3x + 5)(2) - (2x - 11)(3)}{(2x - 11)^2}$
37. The curve  $C$  is given by the equation  $y = 3 \sin x + 2$ . The value of  $\frac{dy}{dx}$  at the point where  $x = \frac{\pi}{3}$  is
- (A)  $\frac{1}{2}$   
 (B)  $\frac{3}{2}$   
 (C)  $\frac{7}{2}$   
 (D)  $3$
38. The point  $P(2, 2)$  lies on the curve with equation  $y = x(x - 3)^2$ . The equation of the normal to the curve at the point  $P$  is given by
- (A)  $y - 2 = 3(x - 2)$   
 (B)  $y - 2 = -3(x - 2)$   
 (C)  $y - 2 = \frac{1}{3}(x - 2)$   
 (D)  $y - 2 = -\frac{1}{3}(x - 2)$

39. The curve  $C$  is given by the equation  $y = 4x + \frac{9}{x}$ . The second derivative,  $\frac{d^2y}{dx^2}$ , is given by

- (A)  $4 + \frac{9}{x^3}$   
 (B)  $\frac{18}{x^3}$   
 (C)  $4 - \frac{9}{x^3}$   
 (D)  $-\frac{9}{2x^3}$

40. The positive value of  $z$  for which

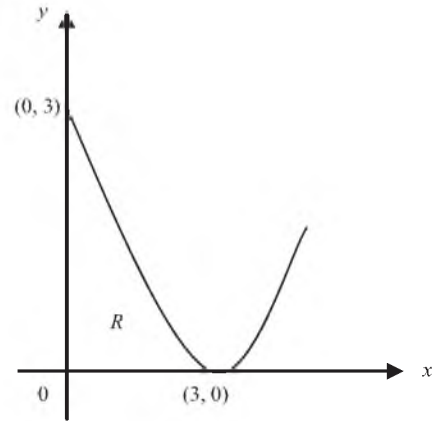
$$\int_0^z x^2 \, dx = 9 \text{ is}$$

- (A) 3  
 (B) 4.5  
 (C) 9  
 (D) 27

41. The gradient of the tangent to a curve  $C$  at  $(x, y)$  is given by  $\frac{dy}{dx} = \frac{1}{(3x + 4)^2}$ . The curve passes through the point  $P\left(-1, \frac{2}{3}\right)$ . The equation of the curve  $C$  is given by

- (A)  $y = \frac{2}{(3x + 4)} + 1$   
 (B)  $y = \frac{-6}{(3x + 4)^3}$   
 (C)  $y = \frac{-2}{3(3x + 4)} + 4$   
 (D)  $y = \frac{-1}{3(3x + 4)} + 1$

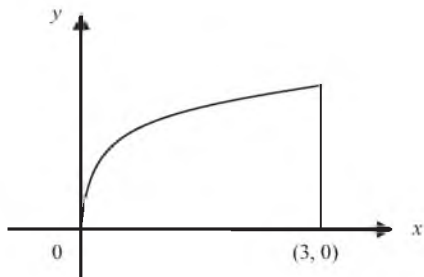
Item 42 refers to the figure below.



42. The finite region  $R$  is bounded by the  $y$ -axis, the  $x$ -axis, and the arc of the curve  $y = (x - 3)^2$  as shown in the figure above. The area of  $R$  in square units is

- (A) 1  
 (B) 3  
 (C) 9  
 (D) 27

**Item 43** refers to the figure below.



43. The finite region enclosed by the curve  $y = \sqrt{x}$ ,  $x \geq 0$ , the  $x$ -axis and the line  $x = 3$ , as shown in the figure above, is rotated completely about the  $x$ -axis. The volume of the solid of revolution formed is given by

(A)  $\int_0^3 \sqrt{x} \, dx$

(B)  $\pi \int_0^3 x \, dx$

(C)  $\pi \int_0^3 \sqrt{x} \, dx$

(D)  $\pi \int_0^3 x^2 \, dx$

44.  $\int (2x + 3)^5 \, dx =$

(A)  $\left[ \frac{1}{6}(2x + 3)^6 \right] + C$

(B)  $\left[ \frac{1}{2}(2x + 3)^6 \right] + C$

(C)  $\left[ \frac{1}{12}(2x + 3)^6 \right] + C$

(D)  $\left[ \frac{1}{3}(2x + 3)^6 \right] + C$

45. Given  $\frac{dy}{dx} = 3 \sin x - 2 \cos x$ , the indefinite integral is given by

(A)  $y = 3 \cos x - 2 \sin x + C$

(B)  $y = -3 \cos x + 2 \sin x + C$

(C)  $y = -3 \cos x - 2 \sin x + C$

(D)  $y = 3 \cos x + 2 \sin x + C$

**END OF TEST**

**FORM TP 2011037/SPEC****C A R I B B E A N   E X A M I N A T I O N S   C O U N C I L****SECONDARY EDUCATION CERTIFICATE  
EXAMINATION****ADDITIONAL MATHEMATICS****SPECIMEN PAPER****Paper 02 – General Proficiency***2 hours and 40 minutes***INSTRUCTIONS TO CANDIDATES**

1. DO NOT open this examination paper until instructed to do so.
2. This paper consists of FOUR sections. Answer ALL questions in Section 1, Section 2 and Section 3.
3. Answer ONE question in Section 4.
4. Write your solutions with full working in the booklet provided.

**Required Examination Materials**

Electronic calculator (non programmable)

Geometry Set

Mathematical Tables (provided)

Graph paper (provided)

2  
SECTION 1

**Answer BOTH questions.**

**All working must be clearly shown.**

1. (a) A function  $f: \mathbb{R} \rightarrow \mathbb{R}$  is defined on domain  $\mathbb{R}$  and co-domain  $\mathbb{R}$ , where  $\mathbb{R}$  is the set of real numbers, by the rule

$$f(x) = x^2$$

- (i) State with reason, whether  $f$  is many-to-one or one-to-one.

**[1 mark]**

- (ii) If instead, the domain of  $f$  is the set of non-negative real numbers,

- a) Determine a function  $g$  such that  $g[f(x)] = x$   
for all values of  $x$  in this domain.

**[1 mark]**

- b) On the same pair of axes, sketch the graphs of  $f$  and  $g$ .

**[2 mark]**

- (b) Use the remainder theorem, or otherwise, to find the remainder when  $x^3 - 2x^2 + 4x - 21$  is divided by  $x - 3$ .

**[2 marks]**

- (c) A student collects laboratory data for two quantities  $q$  and  $p$  as shown in Table 1.

**Table 1**

$q$	1	2	3	4
$p$	0.50	0.63	0.72	0.80

The student reasons a relationship of the form  $p = aq^n$

- (i) Use logarithms to reduce this relation to a linear form.

**[2 marks]**

- (ii) Using the graph paper provided and a scale of 1 cm to represent 0.1 units on the horizontal axis and a scale of 2 cm to represent 0.1 units on the vertical axis, plot a suitable straight line graph and hence estimate the constants  $a$  and  $n$ .

**[6 marks]**

**Total 14 marks**

2. (a) Let  $f(x) = 3x^2 + 12x - 18$ .
- (i) Express  $f(x)$  in the form  $a(x + b)^2 + c$ . **[3 marks]**
- (ii) State the minimum value of  $f(x)$ . **[1 mark]**
- (iii) Determine the value of  $x$  for which  $f(x)$  is a minimum. **[1 mark]**
- (b) Find the set of values of  $x$  for which  $2x^2 + 2 > 5x$ . **[4 marks]**
- (c) Given the series  $\frac{1}{2} + \frac{1}{2^4} + \frac{1}{2^7} + \frac{1}{2^{10}} + \dots$
- (i) show that this series is geometric, **[3 marks]**
- (ii) hence, find the sum to infinity of this series. **[2 marks]**

**Total 14 marks**



## SECTION 2

## Answer BOTH Questions

3. (i) Write the equation of the circle  $C$ , with centre  $(-1, 2)$  and radius  $\sqrt{13}$  units.

[1 mark]

- (ii) Find the equation of the tangent to the circle  $C$  at the point  $P(2,4)$ .

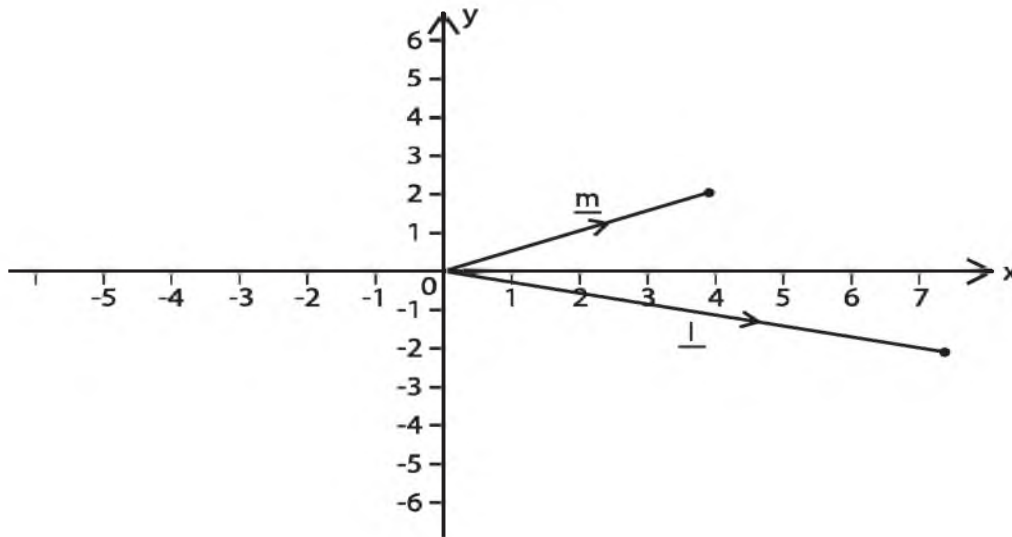
[4 marks]

- (b) The position vector of two points,  $A$  and  $B$ , relative to a fixed origin  $O$ , are  $3t\mathbf{i} + 2t\mathbf{j}$  and  $4\mathbf{i} - 2t\mathbf{j}$  respectively, where  $t > 0$ .

Find the value of  $t$  such that  $\vec{OA}$  and  $\vec{OB}$  are perpendicular.

[4 marks]

- (c) The points  $L$  and  $M$  referred to a fixed origin  $O$  are represented by the vectors  $\mathbf{l} = 7\mathbf{i} - 2\mathbf{j}$  and  $\mathbf{m} = 4\mathbf{i} + 2\mathbf{j}$  respectively, as shown in the diagram below.

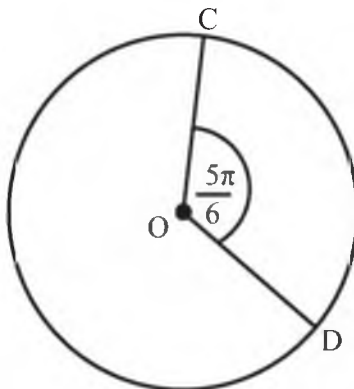


Find the unit vector in the direction of  $\vec{LM}$ .

[3 marks]

Total 12 marks

4. (a) The diagram below shows a circle of centre O and radius 6 cm. The sector COD subtends the angle  $\frac{5\pi}{6}$  at the centre.



Working in **radians**, calculate, giving your answers in terms of  $\pi$ ,

- (i) the length of the minor arc CD [1 mark]
- (ii) the area of the minor sector OCD [2 marks]
- (b) (i) Given that  $\sin \frac{\pi}{4} = \frac{\sqrt{2}}{2}$ , where  $x$  is acute, show that
- $$\sin \left( x - \frac{\pi}{4} \right) = \frac{\sqrt{2}}{2} (\sin x - \cos x).$$
- [2 marks]
- (ii) Using the fact that  $\sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}$  and  $\cos \frac{\pi}{3} = \frac{1}{2}$ , find the exact value of  $\sin \frac{\pi}{12}$  showing ALL steps in your working.
- [3 marks]
- (c) Prove the identity  $\left( \tan \theta - \frac{1}{\cos \theta} \right)^2 \equiv -\frac{\sin \theta - 1}{\sin \theta + 1}$ .
- [4 marks]

**Total 12 marks**

## SECTION 3

## Answer BOTH Questions

5. (a) Differentiate the following with respect to  $x$ , simplifying your result as far as possible

$$(5 - 2x)(1 + x)^4$$

[4 marks]

- (b) The point P lies on the curve  $y = x^2$ . The value of  $x$  at P is -2.

Find the equation of the tangent to the curve at P.

[4 marks]

- (c) Find the stationary points on the curve  $f(x) = 2x^3 - 9x^2 + 12x$  and distinguish their nature.

[6 marks]

Total 14 marks

6. (a) Evaluate  $\int_1^2 (3x - 1)^2 dx$ .

[4 marks]

- (b) Evaluate  $\int_0^{\frac{\pi}{2}} (5 \sin x - 3 \cos x) dx$ .

[4 marks]

- (c) A curve passes through the point P  $\left(0, \frac{7}{2}\right)$  and is such that  $\frac{dy}{dx} = 2 - x$ .

- (i) Find the equation of the curve.

[3 marks]

- (ii) Find the area of the finite region bounded by the curve, the  $x$ -axis, the  $y$ -axis and the line  $x=5$ .

[3 marks]

Total 14 marks

## SECTION 4

Answer Only **ONE** Question

7. (a) In a Lower Sixth Form there are 43 students who are studying either Statistics or Physics or both Statistics and Physics. 28 students study Statistics and 19 students study Physics. If a student is selected at random, what is the probability that he/she is studying

- (i) both Statistics and Physics, **[3 marks]**
- (ii) Physics only. **[2 marks]**

- (b) A tetrahedral die has four faces. Two fair tetrahedral dice, one red and one blue, have faces numbered 0, 1, 2, and 3 respectively. The dice are rolled and the numbers face down on the two dice are recorded.  $R$  is the score on the red die and  $B$  is the score on the blue die.

- (i) Find  $P(R = 3 \text{ and } B = 0)$ .

**[2 marks]**The random variable  $T$  is  $R$  multiplied by  $B$ .

- (ii) Complete the diagram below to represent the sample space that shows all the possible values of  $T$

<b>3</b>				
<b>2</b>		2		
<b>1</b>	0			
<b>0</b>				
B / R	<b>0</b>	<b>1</b>	<b>2</b>	<b>3</b>

**Sample space diagram of T****[3 marks]**The table below gives the probability of each possible value of  $t$ .

$t$	0	1	2	3	4	6	9
$P(T = t)$	$a$	$\frac{1}{16}$	$\frac{1}{8}$	$b$	$c$		

- (iii) Find the values of  $a$ ,  $b$  and  $c$ .

**[3 marks]**

7. (c) The number of cars parked on a local beachfront on each night in August last year is summarized in the following stem and leaf diagram.

1	0 5	
2	1 2 4 8	
3	0 3 3 3 4 7 8 8	Key: 1 0 means 10
4	1 1 3 5 8 8 8 9 9	
5	2 3 6 6 7	
6	2 3 4	

- (i) Find the median and quartiles for these data. **[3 marks]**
- (ii) Construct a box-and-whisker plot to illustrate these data and comment on the shape of the distribution. **[4 marks]**

**Total 20 marks**

8. (a) A car moves along a horizontal straight road, passing two points  $A$  and  $B$ . The speed of the car at  $A$  is  $15\text{ m s}^{-1}$ . When the driver passes  $A$ , he sees a warning sign  $W$  ahead of him, 120 m away. He immediately applies the brakes and the car decelerates uniformly, reaching  $W$  at a speed of  $5\text{ m s}^{-1}$ . At  $W$ , the driver sees that the road is clear. He then immediately accelerates the car with uniform acceleration for 16 secs to reach a speed of  $V\text{ m s}^{-1}$  where  $V > 15$ . He then maintains a constant speed of  $V\text{ m s}^{-1}$  for 22 secs, passing  $B$ .

- (i) Sketch, on the graph paper provided a velocity-time graph to illustrate the motion of the car as it moves from  $A$  to  $B$ . **[3 marks]**
- (ii) Find the time taken for the car to move from  $A$  to  $B$ . **[3 marks]**

The distance from  $A$  to  $B$  is 1 km.

- (iii) Find the value of  $V$ . **[5 marks]**

- (b) A particle moves in a straight line so that  $t$  seconds after passing through a fixed point  $O$ , its velocity,  $v$  m/s is given by

$$v = 3t^2 - 30t + 72.$$

Calculate the:

- (i) values of  $t$  when the particle is at instantaneous rest, **[3 marks]**
- (ii) distance moved by the particle during the interval between the two values of  $t$  found in b (i). **[3 marks]**
- (iii) total distance moved by the particle between  $t = 0$  and  $t = 7$ . **[3 marks]**

**Total 20 marks**

**END OF TEST**

TEST CODE 01254030/SPEC

FORM TP 2011038/SPEC

**C A R I B B E A N E X A M I N A T I O N S C O U N C I L**  
**H E A D Q U A R T E R S**

**SECONDARY EDUCATION CERTIFICATE**  
**EXAMINATION**

**ADDITIONAL MATHEMATICS**

**PAPER 03/2**

**ALTERNATIVE**

*90 minutes*

**Answer the given questions**

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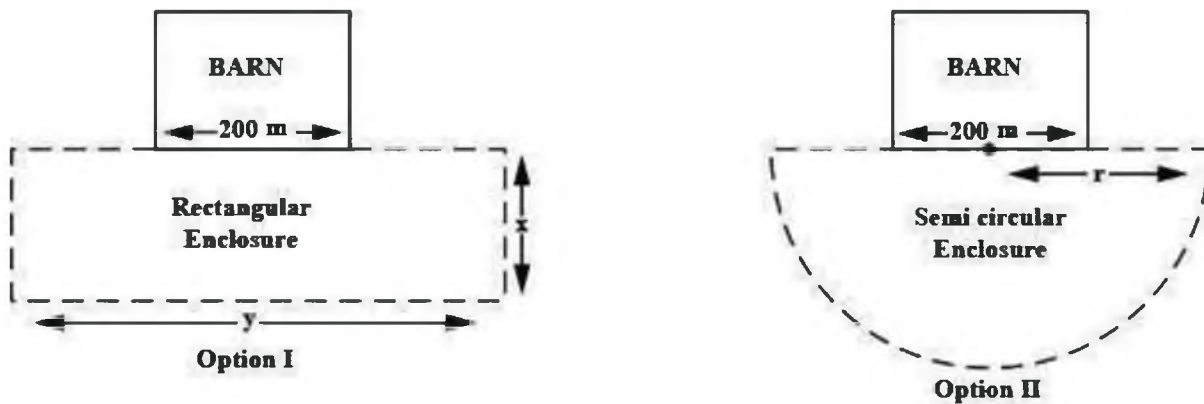
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A farmer plans to construct an enclosure for his goats making use of one side of a barn that is 200 m in length. He has 800 m of fencing material available and wants to maximize the enclosed area.

The farmer must decide whether to make a rectangular or semicircular enclosure as shown in the diagrams below.

You are given that the radius of the semi circular enclosure is  $r$ , the length of the rectangular enclosure is  $x$  and the width is  $y$ .



- (i) Formulate the given real world problem **mathematically**. (7 marks)
- (ii) Show that for Option I a square enclosure maximizes the area and determine the **MAXIMUM** possible area. (7 marks)
- (iii) Determine the **MAXIMUM** area of the enclosure in Option II. (4 marks)
- (iv) Make a recommendation to the farmer on the **MOST** appropriate enclosure, giving a reason. (2 marks)

**Total 20 marks**



## CSEC ADDITIONAL MATHEMATICS SPECIMEN PAPER 01

Item	Key	Specific Objective
1	A	1A4
2	C	1A1
3	B	1F4
4	B	1F9
5	C	1F7
6	C	1F11
7	D	1E3
8	D	1E2
9	B	1E6
10	A	1E5
11	D	1E1
12	C	1B1
13	D	1B4
14	C	1B2
15	A	1D3
16	B	1D4
17	D	1D7
18	D	1D7
19	A	1C1
20	C	1C2
21	B	2A2
22	D	2A3
23	D	2A5
24	A	2B7
25	A	2B10
26	D	2B9
27	B	2C10
28	A	2C10
29	B	2C4
30	B	2C11
31	D	2C11
32	B	2C8
33	A	2C6
34	C	2C11
35	D	3A8
36	C	3A8
37	B	3A5
38	C	3A17
39	B	3A14
40	A	3B8
41	D	3B9
42	C	3B10(i)
43	B	3B10(ii)
44	C	3B5
45	C	3B7

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EXAMINATION**

**ADDITIONAL MATHEMATICS**

**PAPER 02**

**SPECIMEN PAPER  
KEY AND MARK SCHEME**

**ADDITIONAL MATHEMATICS  
PAPER 02  
KEY AND MARK SCHEME**

**Question 1**

- (a) (i)  $f$  cannot be 1-1 since for example  $f(-2) = 4$  and  $f(2) = 4$ . Two objects can have the same images. Thus  $f$  is many-to-one (accept any reasonable explanation or horizontal line test)

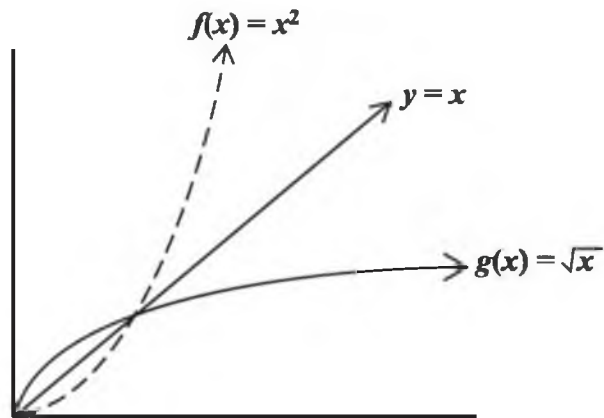
(ii)

- a) If  $g[f(x)] = x$  then  $g$  must be the inverse of  $f$ . Let  $y = x^2$  where  $x$  is a non-negative real number.

$$\text{Then } x = \sqrt{y} = g(x) = \sqrt{x}$$

$$\text{or } = \sqrt{(x^2)} = x, \text{ for all } x \geq 0.$$

b)



- (b) Using  $f(3) = 0$

$$f(3) = 3^3 - 2(3)^2 + 4(3) - 21$$

$$f(3) = 0$$

- (c) (i)  $p = aq^n \Rightarrow \log p = \log a + n \log q$

CK	AK	R
		1
	1	
1	1	
		1
	1	
1		1

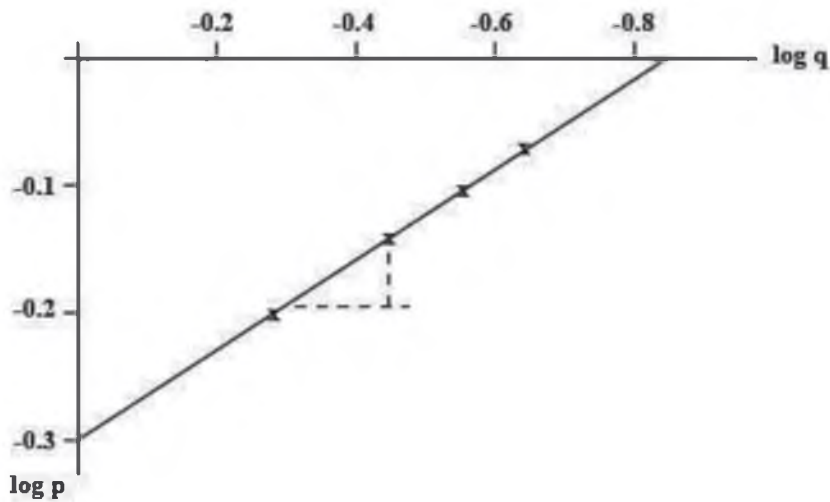
**ADDITIONAL MATHEMATICS  
PAPER 02  
KEY AND MARK SCHEME**

(ii)

Plot  $\log p$  against  $\log q$ : First find the table of values

$\log q$	0	0.3	0.48	0.6
$\log p$	-0.30	-0.20	-0.14	-0.10

On the graph paper provided plot the points and draw a straight line through the points



The  $y$  intercept  $c = -0.3 = \log a$

$$\therefore a = 10^{-0.3} = 0.5$$

The slope of the line is  $\frac{y_2 - y_1}{x_2 - x_1} = m = 0.33$  (by measurement)

$$\therefore n = 0.33$$

(graphical measurements of slope may vary slightly above or below this value)

(Specific Objectives-Sect 1: A3, D3-5, D7, E7-8)

CK	AK	R
1	1	1
3	6	5

**ADDITIONAL MATHEMATICS  
PAPER 02  
KEY AND MARK SCHEME**

**Question 2**

- (a) (i)  $3x^2 + 12x - 18$   
 $3[x^2 + 4x] - 18$   
 $3[(x + 2)^2 - 4] - 18$   
 $3(x + 2)^2 - 12 - 18$   
 $3(x + 2)^2 - 30$
- (ii) Minimum value is  $y = -30$
- (iii) Value of  $x$  at minimum point is  $x = -2$
- (b)  $2x^2 + 2 > 5x$   
 $2x^2 - 5x + 2 > 0$   
 $(2x - 1)(x - 2) > 0$   
 $x < \frac{1}{2}$   
or  $x > 2$
- (c) (i) Given the series  $\frac{1}{2} + \frac{1}{2^4} + \frac{1}{2^7} + \frac{1}{2^{10}} + \dots$   
S is a geometric series if  $\frac{T_n}{T_{n-1}} = r$   
i.e. S has a common ratio  $r$ .
- $$r = \frac{\frac{1}{2^7}}{\frac{1}{2^4}} = \frac{2^4}{2^7}$$
- $$r = \frac{1}{2^3} = \frac{1}{8}$$

CK	AK	R
1	1	1
	1	1
		1
		1
	1	
	1	
1		1
	1	

**ADDITIONAL MATHEMATICS  
PAPER 02  
KEY AND MARK SCHEME**

**Question 2****(cont'd)****(c) (ii)**

$$S_{\infty} = \frac{a}{1-r} = \frac{\frac{1}{2}}{1-\frac{1}{8}}$$

$$S_{\infty} = \frac{\frac{1}{2}}{\frac{7}{8}} = \frac{4}{7}$$

(Specific Objectives - Sect 1: B1, B2, C1, F6, F11)

CK	AK	R
1	1	
<b>3</b>	<b>6</b>	<b>5</b>

**ADDITIONAL MATHEMATICS  
PAPER 02  
KEY AND MARK SCHEME**

**Question 3**

(a) (i)  $(x + 1)^2 + (y - 2)^2 = (\sqrt{13})^2$

$$(x + 1)^2 + (y - 2)^2 = 13$$

(ii) The gradient,  $m$ , of the radius through P (2, 4)

$$\text{is } m = \frac{4 - 2}{2 - 1}$$

$$= \frac{2}{3}$$

The gradient of the tangent to circle P is  $-\frac{3}{2}$

The equation of the tangent at P is given by

$$\frac{y - 4}{x - 2} = -\frac{3}{2}$$

$$2y - 8 = -3x + 6$$

$$2y + 3x = 14$$

(b)  $(3t\mathbf{i} + 2t\mathbf{j}) \cdot (4\mathbf{i} - 2t\mathbf{j}) = 0$

$$12t - 4t^2 = 0$$

$$4t(3 - t) = 0$$

$$t = 0 \text{ or } t = 3$$

$$t = 3, \text{ since } t > 0$$

CK	AK	R
1		
1		
		1
	1	
	1	
		1

**ADDITIONAL MATHEMATICS  
PAPER 02  
KEY AND MARK SCHEME**

**Question 3  
(cont'd)**

(c) 
$$\vec{LM} = \vec{LO} + \vec{OM}$$

$$= -\begin{pmatrix} 7 \\ -2 \end{pmatrix} + \begin{pmatrix} 4 \\ 2 \end{pmatrix}$$

$$= \begin{pmatrix} -3 \\ 4 \end{pmatrix}$$

$$= -3\mathbf{i} + 4\mathbf{j}$$

The unit vector in the direction of  $\vec{LM}$  is

$$\frac{1}{\sqrt{(-3)^2 + (4)^2}} (-3\mathbf{i} + 4\mathbf{j})$$

$$= \frac{1}{5} (-3\mathbf{i} + 4\mathbf{j})$$

(Specific Objectives-Sect 2: A4, A6, B5, B10)

CK	AK	R
1		
1	1	
<b>4</b>	<b>5</b>	<b>3</b>



**ADDITIONAL MATHEMATICS  
PAPER 02  
KEY AND MARK SCHEME**

**Question 4**

- (a) (i)  $l_{\text{CD}} = r\theta$   
 $= 6 \times \frac{5\pi}{6} = 5\pi \text{ cm}$
- (ii)  $\text{Area}_{\text{sector COD}} = \frac{1}{2}r^2\theta$   
 $= \frac{1}{2}(6)^2\left(\frac{5\pi}{6}\right) = 15\pi \text{ cm}^2$
- (b) (i)  $\sin\left(x - \frac{\pi}{4}\right) = \sin x \cos \frac{\pi}{4} - \cos x \sin \frac{\pi}{4}$   
 $= \frac{\sqrt{2}}{2} \sin x - \frac{\sqrt{2}}{2} \cos x$   
 $= \frac{\sqrt{2}}{2} (\sin x - \cos x)$
- (ii)  $\sin\left(\frac{\pi}{12}\right) = \sin\left(\frac{\pi}{3} - \frac{\pi}{4}\right)$   
 $= \frac{\sqrt{2}}{2} \left(\sin \frac{\pi}{3} - \cos \frac{\pi}{3}\right)$   
 $= \frac{\sqrt{2}}{2} \left(\frac{\sqrt{3}}{2} - \frac{1}{2}\right)$   
 $= \frac{\sqrt{2}}{4} (\sqrt{3} - 1)$

CK	AK	R
	1	
1		1
		1
	1	
		1
1		
	1	

**ADDITIONAL MATHEMATICS  
PAPER 02  
KEY AND MARK SCHEME**

**Question 4  
(cont'd)**

4 (c)

$$\text{LHS} \equiv \left( \frac{\sin \theta}{\cos \theta} - \frac{1}{\cos \theta} \right)^2$$

$$\equiv \left( \frac{\sin \theta - 1}{\cos \theta} \right)^2$$

$$\equiv \frac{(\sin \theta - 1)^2}{1 - \sin^2 \theta}$$

$$\equiv \frac{(\sin \theta - 1)^2}{(1 - \sin \theta)(1 + \sin \theta)}$$

$$\equiv \frac{\sin \theta - 1}{\sin \theta + 1}$$

(Specific objectives-Sect 2: C3, C5, C8, C10)

CK	AK	R
		1
	1	
	1	
		1
<b>2</b>	<b>5</b>	<b>5</b>



**ADDITIONAL MATHEMATICS  
PAPER 02  
KEY AND MARK SCHEME**

**Question 6**

(a) 
$$\int_1^2 (3x - 1)^2 dx = \int_1^2 (9x^2 - 6x + 1) dx$$

$$= \left[ 3x^3 - 3x^2 + x \right]_1^2 = (24 - 12 + 2) - (3 - 3 + 1)$$

$$= 13$$

(b) 
$$\int_0^{\frac{\pi}{2}} (5 \sin x - 3 \cos x) dx = [-5 \cos x - 3 \sin x]_0^{\frac{\pi}{2}}$$

$$= [-5(0) - 3(1)] - [-5(1) - 3(0)]$$

$$= 2$$

(c) (i)

$$y = \int (2 - x) dx = 2x - \frac{x^2}{2} + C$$

At  $\left(0, \frac{7}{2}\right)$   $C = \frac{7}{2}$

$$y = 2x - \frac{x^2}{2} + \frac{7}{2}$$

(ii) Area = 
$$\int_0^5 \left( 2x - \frac{x^2}{2} + \frac{7}{2} \right) dx = \left[ x^2 - \frac{x^3}{6} + \frac{7x}{2} \right]_0^5$$

$$\text{Area} = \left( 25 - \frac{125}{6} + \frac{35}{2} \right) = \frac{65}{3} \text{ units}^2$$

(Specific Objectives- Sect 3: B4-5, B7--10)

CK	AK	R
		1
1	1	
	1	
1	1	
		1
	1	
1	1	
		1
1		
	1	
<b>4</b>	<b>6</b>	<b>4</b>

**ADDITIONAL MATHEMATICS  
PAPER 02  
KEY AND MARK SCHEME**

**Question 7**

(a) (i)  $P(\text{Statistics}) = \frac{28}{43}$      $P(\text{Physics}) = \frac{19}{43}$

$$P(\text{Statistics and Physics}) = \frac{28}{43} + \frac{19}{43} - \frac{43}{43} = \frac{4}{43}$$

(ii)  $P(\text{Physics only}) = \frac{19}{43} - \frac{4}{43} = \frac{15}{43}$

(b) (i)  $P(\text{Red} = 3 \text{ and Blue} = 0) = \frac{1}{16}$

(ii)

<b>3</b>	0	3	6	9
<b>2</b>	0	2	4	6
<b>1</b>	0	1	2	3
<b>0</b>	0	0	0	0
<b>B</b> <b>R</b>	<b>0</b>	<b>1</b>	<b>2</b>	<b>3</b>

(iii)

$t$	0	1	2	3	4	6	9
$P(t)$	$\frac{7}{16}$	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{1}{16}$

$$a = \frac{7}{16}, \quad b = \frac{1}{8} \quad \text{and} \quad c = \frac{1}{16}$$

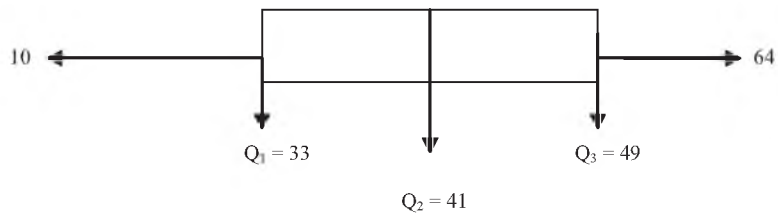
CK	AK	R
1	1	1
	1	1
1	1	
1	1	1
1	1	1

**ADDITIONAL MATHEMATICS  
PAPER 02  
KEY AND MARK SCHEME**

**Question 7  
cont'd**

(c) (i)  $Q_1 = 33$     $Q_2 = 41$     $Q_3 = 49$

(ii)



The shape is symmetrical about the median

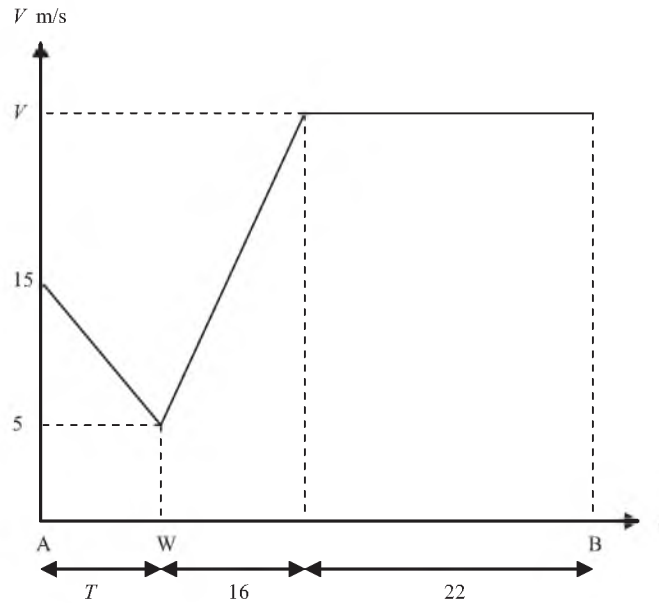
(Specific Objectives- Sect 4: A 4-6, B2-4, B9).

CK	AK	R
1	1	1
1	1	1
	1	
<b>6</b>	<b>8</b>	<b>6</b>

**ADDITIONAL MATHEMATICS  
PAPER 02  
KEY AND MARK SCHEME**

**Question 8**

(a) (i)



(ii) Using area under the curve  $\frac{1}{2}(15 + 5)(T) = 120$

$$T = 12$$

$$\text{Total time} = 12 + 16 + 22 = 50 \text{ secs}$$

(iii) Using area under the curve

$$120 + \frac{1}{2}(V + 5) \times 16 + 22V = 1000$$

$$V = 28$$

CK	AK	R
1	1	1
1		
	1	1
		1
1	1	1
	1	

**ADDITIONAL MATHEMATICS  
PAPER 02  
KEY AND MARK SCHEME**

**Question 8  
(cont'd)**

(b) (i)  $v = 0 \Rightarrow 3t^2 - 30t + 72 = 0$

$$(t - 6)(t - 4) = 0$$

$$t = 4 \text{ secs, } 6 \text{ secs}$$

(ii) Distance =  $\left| \int_4^6 (3t^2 - 30t + 72) dt \right|$  (below the  $x$ -axis)

$$= \left| \left[ t^3 - 15t^2 + 72t \right]_4^6 \right|$$

$$= \left| (216 - 540 + 432) - (64 - 240 + 288) \right|$$

$$= 4 \text{ metres}$$

(iii) Distance in first 4 secs =  $\int_0^4 (3t^2 - 30t + 72) dt$

$$= 112 \text{ metres}$$

Distance between 6 secs and 7 secs

$$= \int_6^7 (3t^2 - 30t + 72) dt$$

$$= 4 \text{ metres}$$

$$\text{Total distance} = 112 + 4 + 4 = 120 \text{ metres}$$

(Specific Objectives- Sect 4: C2 – 4)

CK	AK	R
		1
1	1	
		1
	1	
1		
		1
1		
	1	
	1	
<b>6</b>	<b>8</b>	<b>6</b>



**C A R I B B E A N E X A M I N A T I O N S C O U N C I L**  
**H E A D Q U A R T E R S**

**S E C O N D A R Y E D U C A T I O N C E R T I F I C A T E**  
**E X A M I N A T I O N**

**A D D I T I O N A L M A T H E M A T I C S**

**P A P E R 0 3 / 2**

**S P E C I M E N P A P E R**

**K E Y A N D M A R K S C H E M E**

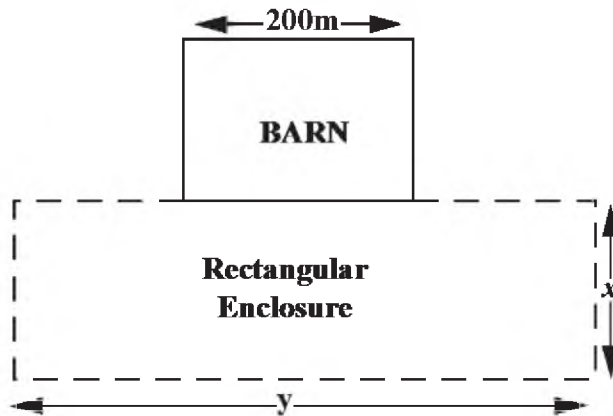
**MATHEMATICS  
PAPER 03  
KEY AND MARK SCHEME**

Question 1

(i)

**Mathematical Formulation of the problem**

**Option I**



Let the rectangular enclosure have length  $x$  m and width  $y$  m. Since one side of the barn of length 200 m is used in making the enclosure and an additional 800m of fencing is available then the perimeter of the enclosure is  $200 + 800 = 1000$  m

The perimeter of the rectangle is  $(2x + 2y)$  m and

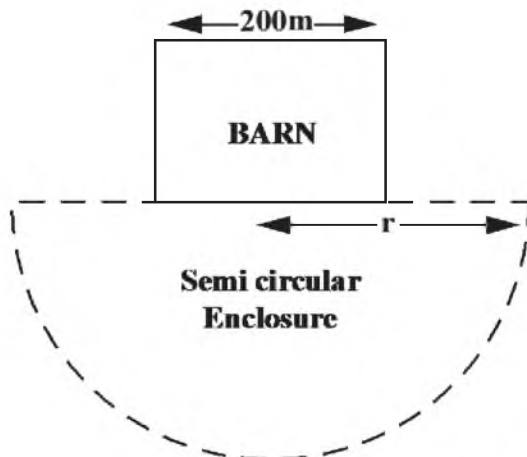
Area of the rectangle enclosure is  $xy$  m<sup>2</sup>

Thus the problem can be formulated mathematically as

Maximise  $A_1 = xy$

Subject to  $2x + 2y = 1000$

**Option II**



CK	AK	R
		1
		1
	1	
	1	

**MATHEMATICS**  
**PAPER 03**  
**KEY AND MARK SCHEME**

Let the semicircular enclosure have radius  $r$

Since half the circumference of a circle is  $\pi r$  and diameter is  $2r$ , the problem is to find the Area  $A_2$  of the semicircle when  $\pi r + 2r = 1000\text{m}$

The solution lies in comparing the two areas Maximum  $A_1$  and  $A_2$  to see which is larger and recommend the larger option to the farmer.

(ii) **To show that for Option I a square maximizes the area:**

Maximise  $A_1 = xy$  (area)

Subject to  $2x + 2y = 1000$  (perimeter)

Solution

$$2x + 2y = 1000 \longrightarrow y = \frac{1000 - 2x}{2} = 500 - x$$

Substituting in  $A_1$  we obtain

$$A_1 = xy = x(500 - x) = 500x - x^2$$

To maximize  $A_1$  with respect to  $x$  we first find the stationary points:

$$\frac{dA}{dx} = 500 - 2x$$

$$500 - 2x = 0 \longrightarrow 2x = 500$$

$\therefore x = 250$  at maximum

(we need not find the 2<sup>nd</sup> derivative as there is only one solution but

$$d^2A/dx^2 = -2 \longrightarrow \text{stationary point is a maximum})$$

$$\therefore y = 500 - x = 250$$

Since  $x = y$  the rectangle is a square

$$\text{Maximum Area is } A_1 = 250^2 = 62\,500 \text{ m}^2$$

**Option II Area**

(iii)

Perimeter of enclosure =  $\pi r + 2r = 1000 \text{ m}$

$$\therefore r = \frac{1000}{\pi + 2} = 194.49 \text{ m (radius of enclosure)}$$

$$\text{Area of semicircle is } A_2 = \frac{\pi}{2} \left( \frac{1000}{\pi + 2} \right)^2$$

This is the maximum area of the semicircle since  $r$  is fixed

$$\text{Area of semicircle is } A_2 = 59\,419 \text{ m}^2$$

CK	AK	R
1		1
		1
	1	
		1
	1	
	1	
		1
		1
1		
	1	

**MATHEMATICS**  
**PAPER 03**  
**KEY AND MARK SCHEME**

**Recommendation**

- (iv) Since  $62500 > 59419$ , the square enclosure has greater area than the semicircular. I would therefore recommend Option I to the farmer, i.e. build a square enclosure of side 250 m

CK	AK	R
		1 1
4	6	10

**CARIBBEAN EXAMINATIONS COUNCIL**

**REPORT ON CANDIDATES' WORK IN THE  
CARIBBEAN SECONDARY EDUCATION CERTIFICATE® EXAMINATION**

**MAY/JUNE 2012**

**ADDITIONAL MATHEMATICS  
GENERAL PROFICIENCY EXAMINATION**

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## GENERAL COMMENTS

Additional Mathematics, the newest CXC subject offering at the CSEC level, was tested for the first time in the May/June 2012 examinations. The subject is intended to bridge a perceived gap between the CSEC General Mathematics and the CAPE Unit 1 Pure Mathematics. The intent is to offer CSEC Mathematics students a more seamless transition to the advanced thinking and skills needed for CAPE Unit 1 Mathematics courses, although a good CSEC Mathematics student should still be able to meet the skills and thinking demands of the CAPE Unit 1 Mathematics course.

The examinations consist of three papers:

- Paper 01 — a 45 – item multiple choice paper;
- Paper 02 — a structured, ‘essay-type’ paper consisting of 8 questions;
- Paper 031 or Paper 032 — Paper 03 represents an School-Based Assessment (SBA) project component for candidates in schools, and Paper 032 an alternative to the SBA for out-of-school candidates.

The Additional Mathematics syllabus (CXC 37/G/SYLL 10) tests content in four main topic areas divided as follows: Section 1 — Algebra and Functions, Section 2 — Coordinate Geometry, Vectors and Trigonometry, Section 3 — Introductory Calculus, and Section 4 — Basic Mathematical Applications.

Paper 01 tests content from Sections 1, 2 and 3 of the syllabus. This paper carries 45 items which are weighted up to 60 for the overall examination. Paper 02 tests content from all four sections of the syllabus. This year the paper consisted of four sections, divided as described previously for the outline of the syllabus. Each section contained two problem-solving type questions. The questions in Sections 1, 2 and 3 were all compulsory. The two questions in these sections were worth a total of 28, 24 and 28 marks respectively. Section 4 also contained two questions, one on Data Representation and Probability and the other on Kinematics worth 20 marks each; candidates were to choose one question from this section. Paper 03 represents the SBA component of the examination. Candidates can do a project chosen from two project types, a mathematical modelling project (Project A) and a data handling/statistical analysis project (Project B). The SBA component is worth 20 marks. Alternatively, candidates can sit an alternative to the SBA paper, Paper 032, which consists of a more in-depth extended question from Sections 1, 2 and/or 3 of the syllabus. This paper carries 20 marks.

## DETAILED COMMENTS

### Paper 01 – Structured Essay Questions

This was a 45 - item paper covering Sections 1, 2 and 3 of the syllabus. A total of 1720 candidates sat this paper of the examination. Candidates scored a mean of 23.22, with standard deviation 8.77.

### Paper 02 – Structured Essay Questions

No formula sheet was provided for this sitting of the examination. However, in cases where it was suspected that candidates might have been affected by the absence of a formula sheet the mark scheme was adjusted so that candidates were awarded follow through marks if they used an incorrect reasonable formula, and were not penalized for this wrong formula. For future examinations, a formula sheet will be provided for papers 01 and 02.

## Section 1: Algebra and Functions

### Question 1

This question tested candidates' ability to:

- determine a composite function, its range and inverse;
- make use of the Factor Theorem;
- use the laws of indices to solve exponential equations in one unknown; and
- apply logarithms to reduce a relationship to linear form.

There were 1746 responses to this question. The mean mark was 5.72 with standard deviation 3.26. Thirty-nine candidates obtained full marks.

Candidates performed best on Parts (a) (i), (iii) and (b) which required them to determine the composite function  $g(f(x))$  of given functions  $f(x)$  and  $g(x)$ , and the inverse of the composite function. In Part (a) (iii) some candidates experienced difficulty at the interchange step, that is,

$$1) \quad (i) \quad g(f(x)) = x^3 + 6; \text{ Let } y = g(f(x)) \quad (ii) \quad \left[ *y^3 = x + 6* \right] \text{ commonly seen error}$$

$$y = \sqrt[3]{x+6}$$

$$\therefore [gf(x)]^{-1} = \sqrt[3]{x+6}$$

Part (b) was generally well done by candidates. However, some of them did have difficulty here. The following common errors were seen:

- Substituting  $x = 2$  into  $f(x)$  rather than  $x = -2$ ;
- Some candidates who did correctly substitute  $x = -2$  then made errors in their manipulation of the directed numbers;
- A few candidates attempted long division approach to solving this problem; however they very often encountered some difficulty with this.

Parts (a) (ii), (c) and (d) presented the most difficulty to candidates. In Part (a) (ii), which required candidates to state the range of the composite function, generally candidates were unable to do this, or stated discrete values for the range, that is,  $\{6, 7, 14, 33\}$ , not recognizing it as the infinite set of Real Numbers,  $6 \leq g(f(x)) \leq 33$ .

In Part (c), many candidates attempted to solve the given equation  $3^{2x} - 9(3^{-2x}) = 8$  without (appropriate) substitution. Having not recognized a suitable substitution to use in order to transform the equation into polynomial form, candidates incorrectly took logarithms of each term in the equation. Some candidates were able to recognize that  $y = 3^{2x}$  or  $y = 3^x$  were suitable substitutions and quite often were able to derive an appropriate polynomial equation. Those who derived the quadratic equation were able to easily solve the indices equation. However, those who derived the quartic equation at times found it challenging to solve. A number of candidates who got to the solution stage of the problem did not recognize (or know) that the logarithm of a negative number did not exist.

In Part (d), although many candidates knew how to transform  $x^3 = 10^{x-3}$  to logarithmic form, they did not use brackets and so wrote  $3 \log_{10} x = x - 3 \log_{10} 10$ , instead of the expected  $3 \log_{10} x = (x - 3) \log_{10} 10$ . Whilst candidates did not lose marks this time given the nature of the specific case here (base is the same as the number so that  $\log_{10} 10$  is 1), teachers are asked to encourage their students to use brackets to avoid

possible errors in the future. Another common error made was having reached to the stage of  $3 \log_{10} x = x - 3$ , a number of candidates then went on to give the gradient as 1, seemingly not recognizing that they had to write the equation in terms of  $\log x$ , that is. to divide through by 3 so that the coefficient of  $\log x$  was 1, i.e.  $\log_{10} x = \frac{1}{3}x - 1$ , so that the gradient of the linear form was  $\frac{1}{3}$ . Other common errors observed were:

- $\text{Log}_{10} x^3 = \log_{10} 10^{x-3}$   
 $\text{Log}_{10} 3x = \log_{10} x - \log_{10} 3$
- $\text{Log}_{10} x^3 = \log_{10} 10x - \log_{10} 30$   
 $3\log_{10} x = 10x - 30$

### Solutions

- (a) (i)  $g(f(x)) = x^3 + 6$  (ii)  $6 \leq g(f(x)) \leq 33$  (iii)  $[g(f(x))]^{-1} = \sqrt[3]{x-6}$   
(b)  $a = 20$   
(c)  $x = 1$   
(d) (i)  $\log_{10} x = \frac{1}{3}x - 1$  (ii) gradient =  $\frac{1}{3}$

### Question 2

This question tested candidates' ability to:

- make use of the relationship between the sums and products of the roots of quadratic equations;
- solve an inequality of the form  $\frac{ax+b}{cx+d} \geq 0$ ;
- identify an AP, obtain an expression for its general term, find its sum to a finite term.

There were 1733 responses to this question. The mean mark was 5.64 with standard deviation 4.27. One hundred and four candidates obtained full marks

Candidates performed best on Part (c) where they had to recognize the problem as an Arithmetic Progression (AP) and find its first term and common difference.

Parts (a) and (b) presented the most difficulty to candidates. In Part (a), many candidates attempted to solve the quadratic equation whether by factorizing or using the formula, showing a seeming lack of knowledge about the sum and product of roots of quadratic equations. In Part (b), although some candidates were able to determine the boundary values for the inequality, they had difficulty with the direction of the inequality for these boundary values which would make the overall inequality 'true'. Although more rarely seen, there was generally more success among candidates who attempted a graphical solution to this question.



In Part (c), a common error observed related to candidates finding the 24<sup>th</sup> term of the AP ( $T_{24}$ ) rather than the sum of the first 24 terms ( $S_{24}$ ). Another common error observed was that some candidates could not correctly write the  $S_n$  formula, commonly writing it as  $S_n = \frac{1}{2} [a + (n - 1)d]$ , instead of the expected  $S_n = \frac{n}{2} [2a + (n - 1)d]$ .

### Solutions

- (a)  $\alpha^2 + \beta^2 = 4$
- (b)  $x > \frac{5}{2}$  and  $x < -\frac{1}{3}$
- (c) Total paid = \$2100

## Section 2: Coordinate Geometry, Vectors and Trigonometry

### Question 3

This question tested candidates' ability to:

- find the centre of a given circle and showing that a given line passes through this centre;
- find the equation of a tangent to a circle at a given point;
- add and subtract vectors, find displacement vectors and find the magnitude of a vector.

There were 1715 responses to this question. One hundred and sixteen candidates obtained full marks.

Candidates performed best on Part (a) (ii) which required them to calculate the gradient of the radius of the circle (from two points), and hence determine the gradient and equation of the tangent to the circle's radius.

Candidates had difficulty with Part (a) (i) which required them to show that the line  $x + y + 1 = 0$  passed through a given circle's centre. Although many could find the coordinates of the circle's centre, some candidates had difficulty in showing (that is, proving) that the given line passed through this centre, indicating a seeming lack of knowledge of what constituted a simple proof in this case. Candidates also had difficulty with Part (b), writing  $\overrightarrow{BP}$  in terms of its position vectors, or correctly adding vectors to get  $\overrightarrow{BP}$ . A common error seen here was that  $\overrightarrow{OP} = \overrightarrow{BP}$ .

### Solutions

- (a) (ii)  $y = \frac{4}{3}x + 11$
- (b) (i)  $\overrightarrow{BP} = \frac{3}{2} \mathbf{a} - \mathbf{b}$                       (ii)  $|\overrightarrow{BP}| = \frac{1}{2}$

#### Question 4

This question tested candidates' ability to:

- use the formulae for arc length and sector area of a circle;
- use the compound angle formula to evaluate a value for  $\sin \theta$  in surd form.

There were 1705 responses to this question. The mean mark was 4.20 with standard deviation 2.24. Two hundred and fifteen candidates obtained full marks.

Candidates performed best on Part (a), that is, finding the area and perimeter of a given shape, although some candidates did not seem to understand the instructions to give their answer in terms of  $\pi$ . For example, some candidates having arrived at the area of the figure as  $16 + \frac{8\pi}{3}$  then went on to give a decimal answer. Other candidates mistakenly simplified this to  $\frac{24\pi}{3}$ . In Part (a) (ii), some candidates failed to recognize that one side of the square (or one radius of the sector) should not be included in the calculation for perimeter, and so added the whole perimeter of the square to the whole perimeter of the sector.

Part (b) presented difficulties for many candidates, in particular coming up with an appropriate split for  $\frac{7\pi}{12}$ ; hence many candidates did not recognize the need for use of the compound angle formula. Some

candidates who recognized the need to split then wrote it as  $\cos \frac{\pi}{3} + \cos \frac{\pi}{4}$  for which no marks were awarded. Of note, a few candidates wrote that  $\cos \frac{7\pi}{12} \equiv \cos \left( \frac{7\pi}{3} + \frac{7\pi}{4} \right)$  and used the fact that  $2\pi$  is one revolution to come to the correct compound angle  $\cos \left( \frac{\pi}{3} + \frac{\pi}{4} \right)$ . The concept of exactness also appeared

to be lost on many of the candidates who proceeded to write the solution,  $\frac{\sqrt{2} - \sqrt{6}}{4}$  as a decimal, or some other incorrect simplification, such as  $-\frac{\sqrt{4}}{4}$ .

Part (c) was not marked. However, approximately  $\frac{1}{3}$  of the candidates who attempted this part of the question knew that  $\sec x = \frac{1}{\cos x}$ .

**Solutions**

(a) (i)  $16 + \frac{8\pi}{3} \text{ m}^2$  (ii)  $16 + \frac{4\pi}{3} \text{ m}$   
(b)  $\frac{\sqrt{2} - \sqrt{6}}{4}$

**Section 3: Introductory Calculus**

Question 5

This question tested candidates' ability to:

- differentiate a quotient made up of simple polynomials;
- determine the equations of tangents and normal to curves;
- make use of the concept of derivative as a rate of change.
- 

There were 1712 responses to this question. The mean mark was 5.89 with standard deviation 4.47. One hundred and thirty-three candidates obtained full marks.

In this question, candidates performed best on the mechanics of differentiation. Many though could not state the correct quotient rule for differentiation, and so performed the mechanics of differentiation in Part (a) as if for a product rule. And, having performed the mechanics of differentiation correctly, a marked number of candidates then could not apply the distributive law correctly with a negative 1 multiplier to expand the brackets, i.e.  $-1(3x + 4)$  was often expanded as  $-3x + 4$ .

Part (b) was generally reasonably done. Some candidates experienced difficulty with the meaning/significance of  $\frac{dy}{dx}$ , in that having found it for the given curve, they could not translate it into finding the value of the gradient at the given point (2, 10). That said, making use of a value for gradient to find the equation of the tangent and the equation of the normal was fairly widely known.

Approximately 50 per cent of candidates did not appear to know the topic 'Rates of Change' and so could not do Part (c). Others, having determined that the area,  $A$ , of the square ( $s$ ) could be written as  $A = s^2$  then went on to differentiate this as  $\frac{dy}{dx} = 2s$  instead of  $\frac{dA}{ds} = 2s$ , showing a lack of understanding of the meaning of differentiation as rate of change. Candidates' difficulty with notation was especially evident in this part of the question; very few candidates wrote their differential equations in terms of  $dt$ .

**Solutions**

(a)  $\frac{-10}{(x-2)^2}$  (b) (i)  $y = 17x - 24$   
(ii)  $10y = x + 172$   
(c)  $\frac{dA}{dt} = 40 \text{ cm}^2 \text{ s}^{-1}$

### Question 6

This question tested candidates' ability to:

- evaluate the definite integral of a function [of the form  $(ax \pm b)^n$ ,  $n \neq -1$ ];
- determine the equation of a curve given its gradient function and a point on the curve;
- find the area of a region bounded by a trigonometric curve and the  $x$ -axis;
- find the volume of revolution of the solid formed when a curve of degree 2 is rotated about the  $x$ -axis.

There were 1655 responses to this question. The mean mark was 6.23 with standard deviation 4.61. One hundred and two candidates obtained full marks.

Candidates performed best on Part (b), although some lost marks as they did not include or failed to find a value for the constant of integration. Part (c) was also relatively reasonably done, although some candidates experienced difficulties in correctly evaluating  $\cos \theta$ . In Part (a), in their integration of  $(16 - 7x)^3$ , although many candidates could perform the mechanics of integration to obtain 4 in the denominator, many candidates failed to multiply the coefficient of  $x$  in the polynomial by the 4 to obtain  $-28$  for the denominator. A few candidates expanded the polynomial in order to integrate it.

Part (d) presented the most difficulty for candidates. Many candidates could set up the problem as  $\pi \int_0^3 (x^2 + 2)^2 dx$ , although some did forget the ' $\pi$ ' in their formula, but a number of candidates were unable to expand  $(x^2 + 2)^2$  correctly, very often obtaining  $x^4 + 4$ .

### **Solutions**

(a)  $\frac{935}{4}$       (b)  $y = \frac{3x^2}{2} - 5x + 4$       (c)  $\frac{2\sqrt{3} + 3}{2}$  units<sup>2</sup>  
(d)  $96.6 \pi$  units<sup>3</sup>

## **Section 4: Basic Mathematical Applications**

### Question 7

This question tested candidates' ability to:

- make use of the laws of probability, specifically the sum of all probabilities in a sample and the addition rule;
- calculate the probability of an event occurring;
- construct and use a tree diagram and calculate conditional probability.

Candidates performed best on Part (a). Many used their knowledge of Sets and Venn Diagrams from CSEC Mathematics to represent the given problem and successfully solve it. However, for candidates who used a formula method to solve, a common error observed was forgetting to subtract the intersection  $P(L \cap D)$  (that is, owned both a laptop and a desktop computer) from the  $P(L) + P(D)$ .

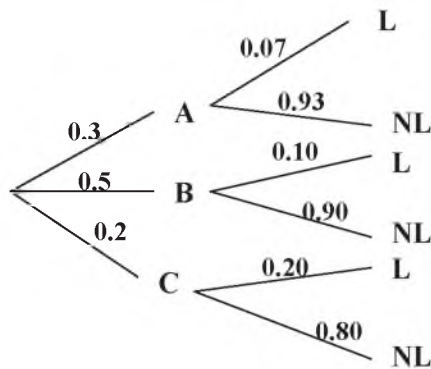
Parts (b) and (c) presented the most difficulty for candidates. In Part (b), a common error observed related to candidates presenting a solution representing sampling with replacement, although the question stated that the items were withdrawn without replacement.

In Part (c) (i), many candidates had difficulty drawing the tree diagram and it was clear that some did not know what this was as they drew diagrams of actual trees, or drew stem and leaf diagrams. Of those who did know what to do, many could only correctly draw the first branch, and had problems completing the second branches for 'late/not late'. Two common errors observed were candidates who correctly wrote the probability of a taxi being late from garage A as  $P(L) = 0.07$  but then calculated  $P(\bar{L}) = 0.23$ , and candidates who incorrectly wrote the probability of a taxi being late from garage A as  $P(L) = 0.7$  instead of  $P(L) = 0.07$  as given. In Part (c) (ii) b, candidates recognized that conditional probability was involved but did not know the formula for conditional probability; they knew that some sort of division was necessary but did not know what to divide by, with many not using the probability they had computed in Part (c) (ii) a) as the denominator here.

**Solutions**

(a) 17%      (b) (i)  $\frac{1}{20}$       (ii)  $\frac{1}{2}$

(c) (i)



(c) (ii) (a) 0.111      (b)  $\frac{40}{111}$

Question 8

This question tested candidates' ability to:

- draw and interpret and make use of velocity-time graphs;
- apply rates of change to the motion of a particle.

There were 879 responses to this question. The mean mark was 6.96 with standard deviation 5.02. Seven candidates obtained full marks.

Candidates performed best on Parts (a) (i) and (ii) which required them to draw a velocity-time graph from given information, and also to determine the distance.

Part (a) (iii) however presented the most difficulty for candidates. This part dealt with a second car starting 3 seconds after the first car from the same starting point, moving at a constant (faster) velocity and meeting the first car sometime later. Most candidates did not know that the distances covered would

have been the same at the meeting point, or that the second car would have taken 3 seconds less than the first car to reach that point. Candidates' experience of difficulty with Part (b) related mainly to notations and their failure to incorporate the constant of integration into their solutions.

### Solutions

- (a) (ii) 264 m (iii) 6 seconds
- (b) (i) velocity =  $32 \text{ ms}^{-1}$  (ii) displacement =  $\frac{52}{3} \text{ m}$

### Paper 031 - School -Based Assessment (SBA)

Generally, many of the projects submitted were of a high quality and related to real-world situations. Many showed a high level of ingenuity, and were obviously scenarios in which students were interested (for example, modelling the trajectory of a basketball to maximize the number of points that could be gained from free throws). A number of the submissions showed evidence that students had applied a high degree of effort to the task, understood the focus of their topic selection and had conducted the requisite research thoroughly. Projects were generally well presented. Marks were generally in the range of 14 to 20, with many of the sample submissions receiving the full score of 20 marks.

There were nonetheless a number of observed weak areas related to sample submissions. These included:

- Some SBA project titles and aims/purpose were not clear, not concise and not well defined. Some titles were too vague, others too wordy. In some cases, students simply restated the project's title as its aim/purpose;
- In a number of cases, the project's aim did not relate to the given title and sometimes not linked to any syllabus objective;
- In some cases for Project B, students did not identify their variables which would be a key aspect of this project type related to the analysis and discussion of the results;
- In some cases for both Projects A and B assumptions were not always stated;
- Some students' data collection method(s) were flawed, with the resulting negative impact on analysis and findings;
- It appeared that all (or most) of the students at some centres conducted their research using the same topic or a limited number of topics. Although there is nothing untoward with this approach, teachers must be aware that the probability of students presenting identical results, analyses and conclusions is very remote. In essence, students must demonstrate individuality in their work;
- In presenting their data or mathematical formulations, many students were short on explanations of what was happening so it was sometimes difficult to follow their presentations;
- Many students had difficulty presenting an appropriate, coherent and full analysis of their data, hence finding(s) and conclusion(s) were deficient;
- The majority of students did not incorporate or suggest any future analysis in their submissions;
- Some students failed to adequately connect their finding(s), discussion and/or conclusion(s) to their original project aim(s)/purpose, their data collected and their analysis of the data. In some cases, the conclusions stated did not arrive out of the presented findings;
- A few submissions appeared to be downloaded from the internet, and there were cases where submitted samples were blatantly plagiarized from CXC/CSEC Additional Mathematics material. Teachers must be alert to any suspected plagiarism. Plagiarism must not be condoned and marked accordingly;

- Some students presented projects that were more applicable to the sciences with an inadequate application of Additional Mathematics concepts;
- Whilst projects were generally well presented, a marked number of them showed deficiencies in grammar, spelling and punctuation.

Other general issues encountered in moderating the SBA samples included:

- There was no CSEC Additional Mathematics rubric/mark scheme accompanying the samples submitted by some centres;
- Some teachers created their own rubric or made adjustments to the CSEC Additional Mathematics rubric;
- Some projects were incorrectly categorized and assessed, that is, a Project A being labelled and assessed as a Project B, and vice versa. This did create some problems as for example, Project B requires the collection and analysis of data from an experimental-type activity.

The following recommendations hold for continued improvement in this aspect of the Additional Mathematics examinations:

- All projects should have a clear and concise title, and well defined aim(s) or purpose;
- Where possible the SBA should identify with authentic situations;
- The variables that are being used or measured (Project B) must be clearly stated and described. Variables can be controlled, manipulated and responding;
- The type of sample and sample size if relevant must be clearly stated;
- Teachers must ensure that projects which integrate other subject areas utilize concepts as contained in the CSEC Additional Mathematics syllabus;
- If students collect their data in a group setting, they must demonstrate their individual effort in relation to analysis (interpretation) and finding(s)/conclusion(s);
- Projects involving dice or playing cards must be more expansive so that students can present a more in-depth analysis of the topic under consideration;
- As good practice, students should be encouraged to cite all sources and insert a reference/bibliography page;
- Teachers should guide students using the assessment criteria found in forms 'Add Math 1 – 5A' and 'Add Math 1 – 5B' which are both available on the CXC website. Teachers can give their students the rubric as a means of guidance in developing their projects.

### **Paper 032 - Alternative to School-Based Assessment (SBA)**

This paper tested candidates' ability to:

- formulate a mathematical problem and derive values to maximize its solution;
- make use of the laws of indices and logarithms to evaluate a specific  $n^{\text{th}}$  value of an AP.

There were 54 responses to this paper. The mean mark was 5.24 with standard deviation 4.22. The highest mark awarded was 18/20.

In Part (a), candidates were able to formulate the mathematical problem as directed. However, a number of candidates lost marks for not finding the second derivative  $\frac{d^2 A}{dx^2}$  (for both sports clubs) and confirming

that it was a maximum. Candidates also made various arithmetic errors in their solutions so few were able to come up with the expected dimensions for both sports clubs.

All candidates who attempted Part (b) approached a solution using the given information that the series was an AP. A number of candidates were able to get one expression for the common difference  $d$  by subtracting two consecutive terms given. However, many failed to obtain a second expression for  $d$  and so could not equate these two to get an expression for  $a$  in terms of  $b$ . Many did not realize that they needed to substitute for  $a$  and so ended up with a final expression in  $ab$ , which, in some cases even though correct, would not have given them the requested value for  $n$ .

### Solutions

- |     |      |                                     |            |  |
|-----|------|-------------------------------------|------------|--|
| (a) | (i)  | Maximize $A = 6xy$                  | Subject to | $9x + 8y = 600$                        |
|     | (ii) | Maximum area $Q = 7500 \text{ m}^2$ | (iii)      | Maximum area $P = 22\,500 \text{ m}^2$ |
| (b) |      | $n = 112$                           |            |  |



*Appendix: Suggested Solutions to Selected Question Parts in Paper 02*

1(c) Solve  $3^{2x} - 9(3^{-2x}) = 8$  [1]  
Let  $3^{2x} = m$ ; Substitute into [1] gives  
$$m - \frac{9}{m} = 8$$
$$m^2 - 9 - 8m = 0$$
$$m^2 - 8m - 9 = 0$$
$$(m - 9)(m + 1) = 0$$
$$m = 9 \text{ OR } m = -1$$
that is,  $3^{2x} = 9$  OR  $3^{2x} = -1$  (INVALID)

From  $3^{2x} = 9$   
 $2x \log 3 = \log 9$   
 $2x = \frac{\log 9}{\log 3} = 2$   
 $x = 1$

2(b)  $\frac{2x - 5}{3x + 1} > 0$   
For LHS  $> 0$ , numerator AND denominator must BOTH have the same sign  
For  $2x - 5 > 0$  AND  $3x + 1 > 0$

$$x > \frac{5}{2} \text{ AND } x > \frac{-1}{2}$$

Both true when  $x > \frac{5}{2}$  [1]

AND for  $2x - 5 < 0$  AND  $3x + 1 < 0$

$$x < \frac{5}{2} \text{ AND } x < \frac{-1}{2}$$

Both true when  $x < \frac{-1}{2}$  [2]

From [1] AND [2], range of values when  $x > \frac{5}{2}$  AND  $x < \frac{-1}{2}$

5(c)  $\frac{ds}{dt} = 4 \text{ cm s}^{-1}$

$$A = s^2; \frac{dA}{ds} = 2s$$

$$\frac{dA}{dt} = \frac{dA}{ds} \times \frac{ds}{dt}$$
$$= 2s \times 4 = 8s$$

When  $s = 5$ ,  $\frac{dA}{ds} = 8 \times 5 = 40 \text{ cm}^2 \text{ s}^{-1}$

7(b) Let R = red marble, B = blue marble, K = black marble

(i)  $P(\text{all of same colour}) = P(RRR) + P(BBB) + P(KKK)$

$$= \frac{4}{10} \times \frac{3}{9} \times \frac{2}{8} + \frac{3}{10} \times \frac{2}{9} \times \frac{1}{8} + \frac{3}{10} \times \frac{2}{9} \times \frac{1}{8}$$

$$= \frac{1}{20}$$

(ii)  $P(\text{exactly one Red}) = P(R\bar{R}\bar{R}) \times 3$

$$= \frac{4}{10} \times \frac{6}{9} \times \frac{5}{8} \times 3$$

$$= \frac{1}{2}$$

7(c) (ii) (a)  $P(L) = P(A \cap L) + P(B \cap L) + P(C \cap L)$   
 $= 0.3 \times 0.07 + 0.5 \times 0.1 + 0.2 \times 0.2$   
 $= 0.021 + 0.05 + 0.04$   
 $= 0.111$

(b)  $P(C/L) = \frac{P(C \cap L)}{P(L)}$

$$= \frac{0.2 \times 0.2}{0.111}$$

$$= \frac{0.04}{0.111}$$

$$= \frac{40}{111}$$

**C A R I B B E A N   E X A M I N A T I O N S   C O U N C I L**

**REPORT ON CANDIDATES' WORK IN THE  
CARIBBEAN SECONDARY EDUCATION CERTIFICATE<sup>®</sup> EXAMINATION**

**MAY/JUNE 2013**

**ADDITIONAL MATHEMATICS  
GENERAL PROFICIENCY EXAMINATION**

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## GENERAL COMMENTS

Additional Mathematics, the newest subject offering at the Caribbean Secondary Education Certificate (CSEC) level, was tested for the second time in the May/June 2013 examinations. The subject is intended to bridge a perceived gap between the CSEC General Mathematics and the CAPE Unit 1 Pure Mathematics. The intent is to offer CSEC Mathematics students a more seamless transition to the advanced thinking and skills needed for CAPE Unit 1 Mathematics courses, although a student who masters the content and skills set out in the CSEC Mathematics syllabus should be ready to meet the skills and thinking demands of the CAPE Unit 1 Mathematics course.

The examinations consist of four papers:

- Paper 01 – a 45-item multiple choice paper
- Paper 02 – a structured, ‘essay-type’ paper consisting of eight questions
- Paper 031 – a School-Based Assessment (SBA) project component for candidates in schools
- Paper 032 – an alternative to the SBA for out-of-school candidates.

Questions in the examination targeted the content and specific objectives of the Additional Mathematics syllabus (CXC 37/G/SYLL 10) with the aim of ensuring appropriate breadth and depth of syllabus coverage.

Paper 01 tests content from Sections 1, 2 and 3 of the syllabus and consists of 45 items. Paper 02 tests content from all four sections of the syllabus. This year the paper consisted of four sections and each section contained two problem-solving type questions. The questions in Sections 1, 2 and 3 were all compulsory. The two questions in these sections were worth a total of 28, 24 and 28 marks respectively. Section 4 also contained two questions, one on Data Representation and Probability and the other on Kinematics, each worth 20 marks. Candidates were required to choose one question from this section. Paper 031 represents the SBA component of the examination. Candidates can do a project chosen from two project types, a mathematical modelling project (Project A) and a data handling/statistical analysis project (Project B). The SBA component is worth 20 marks. Alternatively, private candidates can sit an alternative paper to the SBA, Paper 032, which consists of an in-depth, extended question from Sections 1, 2 and/or 3 of the syllabus. This paper carries 20 marks.

This year saw a 69 per cent increase in candidates registered for the examination, up from 1720 candidates in 2012 to 3100 in 2013.

This year, a formula sheet was included as part of the question paper on Papers 01 and 02.

## DETAILED COMMENTS

### **Paper 01 – Multiple Choice**

This was a 45-item paper covering Sections 1, 2 and 3 of the syllabus. The mean score on this paper was 35.25, with standard deviation of 13.31, compared with 30.96 and 11.69 in 2012.

### **Paper 02 – Structured Essay Question**

This paper consisted of 8 questions, of which questions 1 – 6 were compulsory. Candidates had to choose either Question 7 or Question 8. The total possible score on this paper was 100 marks. The mean and standard deviation for this paper were 47.82 and 27.2 respectively, compared to 37.32 and 22.95 in 2012.

## Section 1: Algebra and Functions

### Question 1

This question tested candidates' ability to:

- make use of the Factor Theorem and factorize a polynomial of degree 3, obtaining all its linear factors;
- determine the inverse of a given function and the composite of two functions;
- use logarithms to solve equations of the form  $a^x = b$ ; simplify expressions using Laws of Logarithms.

There were approximately 2900 responses to this question. The mean mark was 8.02 and the standard deviation was 3.99. Three hundred and forty-four candidates obtained full marks (14 marks).

Candidates performed best on Parts (a) and (b) (ii), with many candidates earning full marks. Part (a) required candidates to show that a given linear expression was a factor of a given polynomial and then go on to find the other linear factors of the polynomial. Most candidates were able to do so either by using the Factor Theorem or via the longer process of long division. More often than not, candidates were then able to successfully factorize the polynomial expression completely. However, the quadratic factor  $x^2 - 5x + 6$  was commonly incorrectly factorized as  $(x - 1)(x - 6)$ . In Part (b) (ii) candidates were required to find an expression for the composite of two functions. Generally, candidates were able to make the correct substitution, although some then made errors in the simplification process. Others still, having correctly simplified and obtained the correct result of  $\frac{2x+1}{x+3}$ , then proceeded to erroneously cancel out the  $x$  in both the numerator and the denominator.

Part (b) (i) and Part (c) presented the greatest difficulty to candidates. In Part (b) (i), which required candidates to obtain an expression for an inverse function, some candidates, having interchanged the variables  $x$  and  $y$ , experienced difficulties in making  $y$  the subject of the formula. Additionally, some candidates did not write their final obtained expression in terms of  $f^{-1}(x)$ , and simply left it in terms of  $y$ . In Part (c), although most candidates knew they needed to take logarithms, they made many errors in applying the mechanics of this to the given problem. For example, in bringing down the indices they did not use brackets, and so made errors in the expansion of the expressions obtained. A common error seen was:

$$\begin{aligned} \lg[5^{3x-2}] &= \lg[7^{x+2}] \\ 3x - 2\lg 5 &= x + 2\lg 7 \\ 2x &= 2\lg 7 + 2\lg 5. \end{aligned}$$

With respect to candidates' responses on this question, there is consolidation needed in the following areas:

- The correct transposition of equations containing more than one variable. For example, transpose for  $y$ ,  $xy + 2x = 2y - 1$ .
- Students must be alert to the fact that when finding  $f^{-1}(x)$ , the final result must contain  $f^{-1}(x)$  equal to some expression in  $x$ .
- Students **must not** consider as equivalent, the *factors of a polynomial expression* and the *roots of a polynomial equation*.
- The correct application of the laws of logarithms. Examples of common misconceptions presented were:  $(\log A)(\log B) = \log A + \log B$ ;  $\log\left(\frac{A}{B}\right) = \frac{\log A}{\log B}$

**Solutions:** (a) (ii)  $(x-3), (x-2)$

(b) (i)  $f^{-1}(x) = \frac{2x+1}{2-x}$  (ii)  $fg(x) = \frac{2x+1}{x+3}$

### Question 2

This question tested candidates' ability to:

- express a quadratic function in the form  $a(x+h)^2 + k$  where  $a$ ,  $h$  and  $k$  are constants; and determine the function's minimum value;
- find the solution set of a quadratic inequality;
- identify geometric series and calculate the sum to infinity of geometric series.

There were 2959 responses to this question. The mean mark was 6.93 and the standard deviation was 3.59. Ninety-one candidates obtained full marks (14 marks).

Candidates performed best on Part (b) of this question, which required them to find the solution set of a quadratic inequality. Most candidates correctly factorized the quadratic and were able to identify the critical values as  $\frac{-5}{2}$  and 1. However, correctly *stating* the solution to satisfy the inequality proved to be a challenge for a number of candidates, even in cases where they were able to correctly identify the regions which satisfied the inequality in a sketch.

Parts (a) and (c) presented some difficulty to candidates. In Part (a), candidates were required to write a given quadratic expression in the form  $a(x+h)^2 + k$ . Approximately 50 per cent of the candidates attempted this question by the method of completing the square, with some candidates encountering difficulties. Some of the candidates used the strategy involving the identity to find the solution as follows:

$$3x^2 + 6x - 1 \equiv a(x+h)^2 + k \rightarrow 3(x+1)^2 - 4 \text{ where } h = 1 \text{ and } k = -4$$

Deductions: (i) Minimum value of function = -4 (ii) Value of  $x$  at minimum = -1

A few candidates attempted to solve for roots of function and abandoned the solution without completing the square.

In Part (c) most candidates were unable to identify two series from the given series, and generally the strategy of separating a given series into two appeared to be unknown to most candidates. Most candidates who attempted the question ignored the note given at the end of the question, and proceeded to find the sum to infinity as if for one series. Very few candidates attempted to separate the series into two GPs. There were also a few candidates who did not seem to know which formula to use to find the sum to infinity, and rather found the sum of the first two terms.

- Solutions:**
- (a) (i)  $3(x + 1)^2 - 4$
- (ii) Minimum value  $f(x) = -4$
- (iii) Value of  $x$  at minimum  $= -1$
- (b) Solution set:  $\{x: x \leq \frac{-5}{2}\} \cup \{x: x \geq 1\}$
- (c)  $\frac{2}{5}$

## Section 2: Coordinate Geometry, Vectors and Trigonometry

### Question 3

This question tested candidates' ability to:

- write the equation of a circle, find the radius of a given circle; and find the equation of a tangent to a circle at a given point;
- apply properties of perpendicular vectors;
- derive a unit vector.

Candidates performed best on Part (a) (i), and Parts (b) and (c) of this question. In Part (a) (i) many candidates were able to write the equation of the circle (without consideration of the radius), and responses of  $x^2 + y^2 - 4x - 2y + K = 0$  or  $(x - 2)^2 + (y - 1)^2 = r^2$  were often seen. In Part (a) (ii) candidates also knew how to find the gradient of the radius and the fact that the tangent is perpendicular to the radius (normal).

In Part (b), a significant number of the candidates either knew that the dot product of perpendicular vectors is zero ( $\overrightarrow{OP} \cdot \overrightarrow{OQ} = 0$ ) or used the formula to find the angles between two vectors and substituted  $90^\circ$  for this angle. In Part (c), the majority of the candidates knew how to find  $\overrightarrow{AB}$ , either from  $\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$  or  $\overrightarrow{AB} = -\overrightarrow{OA} + \overrightarrow{OB}$ . However, they made mistakes in writing down the vectors or in changing the signs, and responses of  $-2i - 5j + 3i - 7j = i - 12j$  or  $2i + 5j + 3i - 7j = 5i - 2j$  were often seen. Candidates also knew how to find the modulus of a vector.

Candidates did have difficulty with part of Part (a) (i), specifically to determine  $r$  or  $K$  in the formula. Many did not seem to know how to do this, or that they needed to do this, and  $(x - 2)^2 + (y - 1)^2 = 0$  was often seen. In Part (b), correctly computing an expression for the dot product proved to be a problem and  $10\lambda + 80$  was often seen. Additionally, some candidates confused the concepts of perpendicularity of lines and perpendicularity of vectors, thus equating the dot product to  $-1$  was seen. In Part (c) finding the unit vector proved to be problematic even in cases where the vector  $\overrightarrow{AB}$  and its modulus were correctly found.

With respect to candidates' responses in this question, the following areas of consolidation are needed:

- Candidates should be taught to derive expressions rather than memorize them. The formula  $\frac{dy}{dx} = \frac{-(g+x)}{f+y}$  was used by some candidates to find the gradient of the tangent; however some of them did not know that  $g$  in their expression was half of  $h$  from the given formula for the equation of a circle and  $f$  was half of  $g$  from the given equation of the circle.

- Candidates should also be encouraged to use techniques appropriate to the topic being tested; for example, in the coordinate geometry question some candidates tried to use calculus to determine the gradient of the tangent. While this technique was a legitimate one, only a few of them were able to correctly do it. The concept of the product of gradients being equal to  $-1$  should be used for perpendicularity of lines in coordinate geometry, while for vectors, equating the dot product to zero should be the preferred method of establishing perpendicularity.
- When the coordinates of the centre of a circle and a point on the circumference are given, candidates should be encouraged to use the formula  $(x - a)^2 + (y - b)^2 = r^2$ , where  $(a, b)$  are the coordinates of the centre and  $r$  is the radius.
- The definition of a unit vector and how to determine a unit in the direction of a given vector should be emphasized.

**Solutions:**

(a) (i)  $x^2 + y^2 - 4x - 2y - 95 = 0$

(ii) Equation of the line l:  $y - 3 = \frac{-4}{3}(x - 10)$

(b)  $\lambda = 8$

(c)  $\vec{AB} = \frac{1}{13}(5\mathbf{i} - 12\mathbf{j})$

#### Question 4

This question tested candidates' ability to:

- use the formulae for arc length and sector area of a circle;
- find solutions of simple trigonometric equations for a given range, which involve the use of  $\cos^2 \theta + \sin^2 \theta \equiv 1$ .
- use the compound angle formula for  $\tan \theta$ .

Candidates performed best on Part (b), that is, solving a given trigonometric equation. Candidates were able to state the necessary trigonometric identity and perform the required substitution, factorize the quadratic equation obtained, identify the range of the trigonometric function and so correctly solve the trigonometric equation for the given range. Some candidates, though, having found the principal angle/value, did have difficulty calculating the other angles that would fit the solution in the given range. Candidates readily drew:

S	A
T	C

but seemed unsure as to how and why this aid was developed. Teachers are urged to explore why each of the trigonometric ratios is positive or negative.

Part (a) presented the most difficulties for candidates. A number of candidates incorrectly stated the formula for the perimeter of a sector, most often only writing this in terms of the arc length and not including  $2r$ . Many then attempted to equate coefficients in finding a value for  $r$  instead of using an algebraic method, and so did not obtain a correct value for  $r$ , nor the sector's area, as asked for in the question. In Part (c), whilst



most candidates were able to identify and use  $\frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \equiv \tan (A \pm B)$ , some candidates had difficulty interpreting that the  $\pm$  and the  $\mp$  signs, and so often wrote:  $\tan (A - B) = \frac{\tan A - \tan B}{1 - \tan A \tan B}$

Teachers are encouraged to have students learn the formulae and to not rely solely on the formula sheet.

**Solutions:** (a) Area =  $\frac{25\pi}{12} \text{ m}^2$

(b)  $\theta = 210^\circ$  and  $330^\circ$

(c)  $\alpha = 45^\circ$

### Section 3: Introductory Calculus

#### Question 5

This question tested candidates' ability to:

- differentiate a simple polynomial made up of  $x^n$ , use the concept of stationary points; calculate the second derivative of polynomials; interpret the significance of the sign of the second derivative; and use the sign of the second derivative to determine the nature of stationary points;
- differentiate  $\sin x$ , differentiate products of simple polynomials and trigonometric functions; apply the chain rule in the differentiation of composite functions.

In this question candidates performed best on Part (a) (i) and well on Part (a) (ii). In Part (a) (i) the concept of stationary points appeared to be familiar to most candidates, and approximately 70 per cent were able to successfully identify the coordinates of the stationary points. Incorrect solutions were obtained by candidates who equated the function rather than the first derivative to zero to obtain the coordinates of the stationary points. Common errors in algebraic manipulations involved solving  $3x(x - 2) = 0$ , and writing  $3x = 0$ ,  $x = -3$ . In Part (a) (ii) most candidates were able to correctly find the second derivative and use the  $x$ -values from Part (a) (i) to deduce the nature of the stationary points. However, a common error seen was equating the second derivative to 0 and solving for  $x$  in order to determine the nature of the stationary points.

Part (b) presented the most difficulty to candidates. It required them to simultaneously use the product and the function of a function rule in differentiating a combination trigonometric and polynomial expression. A number of candidates stopped at one term of the product rule and differentiation of the term in  $x$  in the brackets was not done. Additionally, many candidates incorrectly stated the derivative of  $\sin x$  as  $-\cos x$ . Finally, some candidates had problems simplifying the expression obtained, with a number of them attempting to expand the algebraic term as simplification of the result.

**Solutions:** (a) (i) Coordinates stationary points = (0, 2) and (2, -2)

(ii)  $\frac{d^2y}{dx^2} = 6x - 6$ ; (0, 2) is a maximum point, (2, -2) is a minimum point

(b)  $(5x + 3)^2 [(5x + 3) \cos x + 15 \sin x]$

Question 6

This question tested candidates' ability to:

- use the rules of simple integration;
- integrate simple trigonometric functions and compute definite integrals;
- formulate the equation of a curve, given its gradient function and points on the curve; find the area of a finite region in the first quadrant bounded by a curve and the  $x$ - and  $y$ -axes;

There were 2800 responses to this question. The mean mark was 6.16 and the standard deviation was 4.67. Three hundred and nineteen candidates obtained full marks (14 marks).

Part (a) presented the most difficulty to candidates. In Part (a) a number of candidates did not remember to include the arbitrary constant in integrating the given function. In Part (c) some candidates could not obtain the correct equation of the curve. Even among those who knew to integrate the gradient function to obtain this equation, some did not include a constant of integration and hence obtained an incorrect equation. Additionally, a minority had difficulty obtaining the limits between which they should integrate.

**Solutions:**

(a)  $\frac{5}{3}x^3 + 4x + C$

(b)  $-2$

(c) Area =  $26\frac{2}{3}$  units<sup>2</sup>

**Section 4: Basic Mathematical Applications**Question 7

This question tested candidates' ability to:

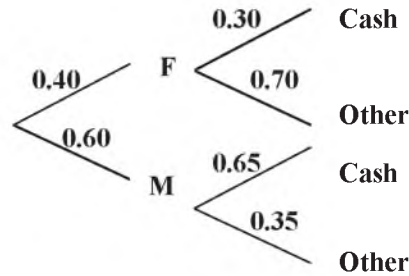
- construct and use a tree diagram to solve problems involving probability; calculate conditional probability; calculate classical probability;
- state the relative advantage of a stem and leaf diagram versus a box and whisker plot to display data; construct a stem and leaf diagram; determine quartiles and measures of central tendency from stem and leaf diagram.

Candidates performed best on Part (a) (i) of this question, which required them to complete the construction of a tree diagram. Candidates knew that the sum of each branch equates to 1. However, having drawn the tree diagram, many candidates did not use it to answer Parts (a) (ii) and (iii). Part (a) (ii), on conditional probability, was badly done, and it appeared that most candidates did not even realise that the question required the use of conditional probability. In Part (a) (iii), most candidates were able to get the probability for Event V, and also to say which was the more likely event.

Parts (b) (i), (ii), (iv) and (v) presented the most difficulty to candidates. In Part (b) (i), which asked candidates to state an advantage of a stem and leaf diagram versus a box and whiskers plot to display the given data, a number of candidates stated their personal preference rather than an advantage, and wrote, for example, 'the stem and leaf is easier to draw and understand'. In Part (b) (ii), which required candidates to construct the stem and leaf diagram, a number of candidates drew tree diagrams. A number of candidates who did draw a stem and leaf diagram lost a mark for not including a key to the diagram. In Part (b) (iv), candidates did not know the formula for semi-interquartile range, or did not know how to work this out. In

many cases they could not correctly find the lower and upper quartiles, and many did not know to divide this difference by two. In Part (b) (v), some candidates mistook the sample space for the marks. Many other candidates did not recognize the sample space being reduced, and so could not correctly give the probability of the second student scoring less than 50 marks on the exam.

**Solutions:** (a) (i)



(ii)  $P(\text{Cash}) = 0.51$

(iii) Event T more likely

(b) (i) All data values are retained; shape of distribution easily seen

(ii)

Stem	Leaf
4	0 1 5
5	0 0 1 1 3 6 6 8 8
6	3 3 6 6 9
7	2 4 5 5 6
8	0 1 3 5 9
9	2 4 9

Key: 6 | 3 means 63

(iii) Median = 66

(iv) Semi-interquartile range = 13.5

(v)  $P(\text{less than 50 marks}) = \frac{3}{30} \times \frac{2}{29} = \frac{3}{435}$

### Question 8

This question tested candidates' ability to:

- draw, interpret and make use of velocity-time graphs;
- calculate and use velocity, acceleration and time in simple equations representing the motion of a particle in a straight line.

Candidates performed best on Parts (a) (i), (b) (i) and (iii) of this question, which required them to draw a velocity-time graph from given information; obtain from its velocity function,  $t$ -values when a particle is at instantaneous rest and obtaining values for  $\frac{dv}{dt}$  at particular points in time.

Parts (a) (ii), (b) (ii) and (iv), however, presented the most difficulty to candidates. In Part (a) (ii), candidates were required to find the total distance travelled by the particle from their graph, as well as to find the average velocity of the particle. A few candidates did not use the graph and used instead the formula, distance = speed  $\times$  time, which gave incorrect values. This formula will only give correct distances if the speed is constant. Other candidates had some problems correctly finding the area under the graph of all parts of the graph. A few candidates did not know how to calculate average velocity, that is, average

velocity =  $\frac{\text{distance}}{\text{time}}$ , with some simply dividing their total distance by 2.

In Part (b) (ii), which required candidates to obtain the distance travelled by the particle between  $t = 1$  and  $t = 3$  by integrating  $v$  to obtain displacement. Whilst a number of candidates correctly obtained that at  $t = 1$ , displacement = 7, and at  $t = 3$ , displacement =  $-9$ , about one-half of them did not know that the distance between  $t = 1$  and  $t = 3$  is 16, as the displacement of  $-9$  meant 9m (or whatever unit of length) to the other side of 0. They experienced difficulty in interpreting distance from the calculated displacement values. In

Part (b) (iv), the interpretation of the  $\frac{dv}{dt}$  values presented difficulty to about one-half of the candidates, with many, for example, believing that  $\frac{dv}{dt} = 0$  implied that the particle was at rest.

- Solutions:**
- (a) (ii) a) 900 m
  - b) average velocity =  $25.7 \text{ m s}^{-1}$
  - (b) (i)  $t = 1$  or  $t = 5$
  - (ii) distance travelled = 16 m (units)
  - (iii) a) at  $t = 2$ ,  $\frac{dv}{dt} = -6$
  - b) at  $t = 3$ ,  $\frac{dv}{dt} = 0$
  - (iv) a) at  $t = 2$ , particle is decelerating
  - b) at  $t = 3$ , acceleration is 0, or constant velocity

### Paper 031 – School Based Assessment (SBA)

Many of the projects submitted were of a high quality and generally related to real-world situations. Many of the submissions received full marks.

The following were general observations on both Projects A and B:

- Some SBA project titles and aims/purpose were not clear, not concise and not well defined. Some titles were too vague, others too wordy.
- In far too many cases, candidates simply re-stated the project's title as its aim/purpose. This often meant that the reason for the project was not stated.
- In a number of cases, the project's aim/purpose did not relate to the given title and sometimes was not linked to any syllabus objective;
- Some submissions did not come with the required CSEC Additional Mathematics rubric;

- Some teachers created their own rubric or made adjustments to the CSEC Additional Mathematics rubric. This is unacceptable, as the project then has to be re-marked using the appropriate CSEC Additional Mathematics rubric;
- A few project submissions appeared to have been downloaded from the Internet;
- There were too many instances of incorrect spelling and grammar. However, candidates used the correct mathematical jargon and symbols appropriately most of the time;
- Some projects were incorrectly categorized and assessed, that is, a Project A being mislabelled and assessed as a Project B, and vice versa. This did create some problems as, for example, Project B requires the collection and analysis of data from an experimental-type activity.

### **Specifically for Project A:**

#### **Mathematical Formulation**

- In stating how the project was going to be done generally most candidates correctly identified all the important elements of the problem and showed understanding of the relationship between the elements.
- The content used in some cases was up only to a Form 3 (or Grade 8) level Mathematics, for example some projects had only simple calculations with area and volume.

#### **The Problem Solution**

- Assumptions were often not clearly stated. Some candidates, though, did state limitations of the project.
- Explanations were generally not sufficient and also were not placed between every successive step in the problem solution.
- Calculations were often precise but solutions were not clearly stated.

#### **Application of Solution**

- Many candidates were unaware of how to show that the solution or proof of the given problem was valid.

#### **Discussion of Findings/Conclusion**

- In some instances the discussion was worthwhile. However, the discussion was not always related to the project's purpose and was often quite broad.
- Conclusions in many instances were not found to be valid. Some candidates stated many conclusions which were not related to the purpose of the project.
- There were only few instances where suggestions for future analyses were stated.

**Specifically for Project B:****Method of Data Collection**

- Although candidates indicated what they did, some were not specific enough about the type of data collected and method of sampling.

**Presentation of Data**

- Candidates had the minimum requirement of one table or chart. However, some tables and charts were not properly named or labeled.
- There was not a systematic layout or organization of the tables and charts.

**Mathematical Knowledge (analysis of data)**

- Mathematical concepts in many cases were not used appropriately from the Additional Mathematics syllabus.
- In many cases candidates used concepts and representations up to just Form 2 level Mathematics; for example, some candidates only used pie charts and bar charts in their representation of the data.
- Most of the calculations shown were accurate. However, in some cases no working was seen because some candidates used software to generate the values.
- Generally some analysis was attempted, but the analysis was often not coherent. This could be attributed to the fact that no proper summary table of calculation values was seen, so the markers had to search through the document constantly to link the calculated values with the analysis.
- The candidates were not too clear on two approaches to be used in the analysis. In many cases the two approaches used were of the same concepts; for example, mean, mode and median were presented as different approaches to the analysis, but all of these are measures of central tendency.

**Discussion of Findings/Conclusions**

- In most instances there was no statement of findings.
- Conclusions made were based on the reported findings but often were not related to the purpose of the project. As a result their conclusions were invalid.
- In most cases there were no suggestions for future analysis.

**Recommendations**

The following recommendations hold for continued improvement in this aspect of the Additional Mathematics examinations:

- Teachers are reminded not to use projects taken directly from the specimen papers or from the exemplar projects given in the Additional Mathematics syllabus.
- In the cases where group work is done, candidates are reminded to submit individual reports.
- All projects should have a clear and concise title, and well-defined aim(s) or purpose;
- Where possible the SBA should be related to authentic situations;

- The variables that are being used or measured (Project B) must be clearly stated and described. Variables can be controlled, manipulated and responding;
- The type of sample and sample size, if relevant, must be clearly stated;
- Teachers must ensure that projects which integrate other subject areas utilize concepts as contained in the CSEC Additional Mathematics syllabus;
- If candidates collect their data in a group setting, candidates **must** demonstrate their **individual** effort in relation to analysis (interpretation) and finding(s)/conclusion(s);
- Projects involving dice or playing cards must be more expansive so that candidates can present a more in-depth analysis of the topic under consideration, for example, this could include multiple and non-standard dice or multiple decks of cards;
- As good practice, candidates should be encouraged to cite all sources and insert a reference/bibliography page;
- Teachers should guide students using the assessment criteria found in forms 'Add Math 1 – 5A' and 'Add Math 1 – 5B' which are both available on the CXC website. Teachers can give their students the rubric as a means of guidance in developing their projects;
- Overall, it is clear that training workshops are needed in the area of project work for Additional Mathematics. Therefore, it is recommended that workshops be held throughout the Caribbean. Teachers need to be aware that there are TWO RUBRIC SHEETS 1-5A for project A (the mathematical modelling project) and 1-5B (the statistical project), and to use the appropriate sheet in marking students' projects. It was apparent that some teachers found difficulties in understanding some aspects of the rubric, or were unaware of the existence of the rubric. Teachers need to ensure that the content level in the project coincides with the Additional Mathematics syllabus.

### Paper 03/2 – Alternative to the School Based Assessment (SBA)

This paper tested candidates' ability to:

- write the equation of a circle; find the points of intersection of a curve with a straight line;
- use the concept of stationary points; differentiate simple polynomials and quotients of simple polynomials.

In Part (a), use of the formula for a circle whose centre was the origin was generally known. Part (b) (i), which required candidates to find an expression for the height of a cuboid given its length, width and volume, as well as Part (b) (ii), which required candidates to write an expression for the total surface area of the cuboid, appeared to be widely known. The formula for total surface area,  $2 \times (l \times w + l \times h + w \times h)$  was also well known.

In Part (a), candidates had difficulty relating the sides of the square to their equations  $x = \pm 5$  and  $y = \pm 5$  and hence did not know they could use simultaneous equations to obtain the other coordinates. In Part (b), candidates for the most part did not know that the stationary point occurred when  $\frac{dA}{dx} = 0$  and many of them

could not differentiate the expression  $\frac{2160}{x}$ .

Teachers need to emphasize that a stationary point occurs when the gradient function,  $\frac{dy}{dx} = 0$ . Teachers also need to do more examples on differentiating functions of the form  $\frac{a}{x^n}$ , where  $a$  is a constant and  $n$  a positive integer.

**Solutions :**

(a)  $(5, \sqrt{11}); (5, -\sqrt{11}); (-5, \sqrt{11}); (-5, -\sqrt{11});$   
 $(\sqrt{11}, 5); (\sqrt{11}, -5); (-\sqrt{11}, 5); (-\sqrt{11}, -5)$

(b) (i)  $h = \frac{360}{x^2}$

Generally, teachers need to ensure that the syllabus is completed with sufficient time to give candidates time to do practice exercise and past papers.



**C A R I B B E A N      E X A M I N A T I O N S      C O U N C I L**

**REPORT ON CANDIDATES' WORK IN THE  
CARIBBEAN SECONDARY EDUCATION CERTIFICATE® EXAMINATION**

**MAY/JUNE 2014**

**ADDITIONAL MATHEMATICS  
GENERAL PROFICIENCY EXAMINATION**

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## GENERAL COMMENTS

Additional Mathematics was tested for the third time in the May/June 2014 examinations. The subject is intended to bridge a perceived gap between the CSEC General Mathematics and the CAPE Unit 1 Pure Mathematics.

The intent is to offer CSEC Mathematics students a more seamless transition to the advanced thinking and skills needed for CAPE Unit 1 Mathematics courses, although a student who masters the content and skills set out in the CSEC Mathematics syllabus should be ready to meet the skills and thinking demands of the CAPE Unit 1 Mathematics course.

The examinations consist of four papers:

- Paper 01 – a 45-item multiple choice paper
- Paper 02 – a structured, ‘essay-type’ paper consisting of eight questions
- Paper 031 – a School-Based Assessment (SBA) project component for candidates in schools
- Paper 032 – an alternative to the SBA for out-of-school candidates.

Questions in the examination targeted the content and specific objectives of the Additional Mathematics syllabus (CXC 37/G/SYLL 10) with the aim of ensuring a wide coverage of the syllabus. The questions were designed at the appropriate level to test the following skills: Conceptual Knowledge, Algorithmic Knowledge and Reasoning.

Paper 01 tested content from Sections 1, 2 and 3 of the syllabus and consisted of 45 items. Paper 02 tested content from all four sections of the syllabus. This year the paper consisted of four sections and each section contained two problem-solving questions. The questions in Section 1, 2 and 3 were all compulsory, and the questions in Section 1 were worth 14 marks each, those in Section 2 were worth 12 marks each and those in Section 3, 14 marks each. Section 4 contained two questions, one on Data Representation and Probability and the other on Kinematics and each question was worth 20 marks. Candidates were required to answer only one question from this section. Paper 031 is the SBA component of the examination. Candidates are required to complete one project chosen from two project types; a mathematical modelling project (Project A) and a data handling/statistical analysis project (Project B). The SBA component is worth 20 marks. Private candidates can sit an alternative paper to the SBA, Paper 032, which consists of one in-depth, extended question from Sections 1, 2 and/or 3 of the syllabus. This paper is worth 20 marks.

This year saw a 16 per cent increase in candidates registered for the examinations, up from 3100 candidates in 2013 to 3597 in 2014.

Again this year, a formula sheet was included as part of the question paper on Papers 01, 02 and 03/2.

## DETAILED COMMENTS

### Paper 01 – Multiple Choice

This was a 45 item paper covering Sections 1, 2 and 3 of the syllabus. The mean score on this paper was 39.41, with standard deviation of 12.71 respectively, compared to 35.25 and 13.31 in 2013.

### Paper 02 – Structured Essay Questions

This paper consisted of eight questions, of which Questions 1 to 6 were compulsory. In addition, candidates had to answer either Question 7 or Question 8. The total possible score on this paper was 100 marks. The mean and standard deviation for this paper were 46.32 and 26.01 respectively, compared to 47.82 and 27.27 in 2013.

## Section 1: Algebra and Functions

### Question 1

This question tested candidates' ability to:

- determine whether a given function is one-to-one; find the composite of two functions and determine its domain; determine the inverse of a given function and sketch the given graph and its inverse on the same pair of axes;
- apply the Remainder Theorem to evaluate an unknown coefficient;
- convert a real-world worded problem to obtain a quadratic equation and solve it in order to find the dimensions of a room.

There were 3 462 responses to this question. The mean mark was 7.50 with standard deviation 3.95. Approximately 5 per cent of candidates obtained full marks.

Candidates performed best in Parts (b) and (c) of this question, with many candidates earning full marks for these parts of the question.

Part (b) required candidates to evaluate the unknown coefficient of  $x$ . Most candidates successfully applied the Remainder Theorem to find this coefficient. Generally candidates were able to make the correct substitution and equate this to the remainder; however some candidates made errors while simplifying the expression.

In Part (c) candidates were required to formulate equations representing the length and width of the room. Most candidates then solved these equations simultaneously to find the required values. However, some candidates used trial and error to arrive at their answers, which at times were correct. Generally many candidates recognised that the positive solution from the quadratic equation was the correct value to be used to find the required length; however some used both positive and negative values to obtain the length without indicating which pair was valid.

Part (a) (i) and (a) (ii) presented the greatest difficulty to candidates.

Part (a) (i) required candidates to show that a given function was NOT one-to-one. Few candidates were able to show that the function was not one-to-one although some simplified the function in a cubic expression correctly using the difference of squares method.

Candidates who obtained full marks were able to do so by either showing that  $f(-a) = f(a)$  for  $a \in \mathbb{R}$  or making a sketch of the function and applying the horizontal line test. Most candidates, when doing Part (a) (ii) a), were able to correctly find the value of the composite function. However, they were unable to state the domain and when they did, it was done incorrectly.

Part (a) (ii) b) also presented some difficulty to candidates. The inverse of the function was at times incorrectly stated; for example, it was often represented as  $2x + 3$ . Many candidates also did not even attempt to sketch the two graphs  $g(x)$  and their inverse in this part of the question, even though they performed well in other parts of the question.

With respect to candidates' responses on this question, consolidation is needed in the following areas:

- More graph work and analysis of graphs need to be done. Students should be given extensive practice in drawing graphs or, in cases where a sketch is required, in representing the data in an efficient and effective way. They also need to differentiate between a quadratic graph and a linear graph.
- Students must not consider as equivalent, the quotient of a polynomial expression and the remainder of a polynomial expression as it applies to the Remainder Theorem.

- Students should be shown the importance of applying mathematical solutions to real-world situations. For example, they correctly found the length and width of the room but did not state these values to represent such. This occurrence had little impact here since once the correct values were seen they were credited. However, if these values were to be used for further analysis of the question some difficulty might have arisen.

**Solutions are:** (a) (i)  $f(-a) = f(a)$  thus  $f$  is many- to- one

(ii) a)  $-\frac{1}{4}x^2 + 3x - 8 \quad x \in \mathfrak{R}$

b)  $g^{-1}(x) = 2(x + 3) \quad x \in \mathfrak{R}$

(b)  $a = 3$

(c) width  $x = 6$  cm and length  $y = 18$  cm

## Question 2

This question tested candidates' ability to:

- express a quadratic function  $f(x)$  in the form  $k + a(x + h)^2$  where  $a$ ,  $h$  and  $k$  are constants; determine the function's maximum value and the value of  $x$  for which the maximum occurred;
- find the solution set of a quadratic inequality;
- identify a geometric series and calculate the sum to infinity of the geometric series.

There were 3 522 responses to this question. The mean mark was 8.19 with standard deviation 4.27.

Candidates performed best in Part (c) of this question. However approximately 20 per cent of the candidates incorrectly used the common ratio as 10. To show that the series was geometric approximately 25 per cent of the candidates used the first two terms only. A few even treated the series as an arithmetic progression and obtained a common difference of 0.18 between the first two terms.

Parts (a) and (b) were equally challenging to the candidates. In Part (a) (i) candidates demonstrated difficulty in factorising from the given function and in completing the square correctly. Most of the candidates used the alternative approach in which they recalled expressions for the required  $h$  and  $k$  based on their previous knowledge. Some candidates made errors in recalling these expressions and the majority of candidates who were able to present a correct expression made errors in computing the numerical values of  $h$  and  $k$ . In Part (a) (ii) most candidates were able to deduce their correct maximum value however some candidates provided a minimum value of  $f(x)$  although the question was clearly stated. Some candidates provided the coordinates of the maximum point instead of providing the function's maximum value. In Part (a) (iii) some candidates did not see the connection of this question to their response in Part (a) (i).

Some of the candidates could not obtain the correct set of values of  $x$  although they correctly factorised and determined the correct critical values. A few candidates failed to change the sign of the inequality when they multiplied by  $-1$ . For the most part, however, the attempt of factorisation was well done.

This question was generally well done by the majority of candidates.

**Solutions are:** (a) (i)  $9 - 2(x + 3)^2$   
(ii) Maximum value  $f(x) = 9$   
(iii) Value of  $x$  at maximum =  $-3$

(b) Solution set:  $\{x: x \leq \frac{-1}{2}\} \cup \{x: x \geq 3\}$

(c) (i) common ratio of  $\frac{1}{10}$

(ii)  $\frac{2}{9}$

## Section 2: Coordinate Geometry, Vectors and Trigonometry

### Question 3

This question tested candidates' ability to:

- apply the perpendicular property for two lines; determine the equation of a circle, given its radius, and whose centre is the point of intersection of two given lines;
- apply properties of perpendicular vectors and determine the magnitude of a displacement vector.

Candidates demonstrated good recall of knowledge of Coordinate Geometry and vectors.

In Part (a) (i) candidates correctly expressed the condition for two straight lines perpendicular to each other as  $m_1 \times m_2 = -1$ ,  $m_2 = -1/m_1$  or the equivalent in words.

In Part (a) (ii) candidates correctly recalled the equation of the circle in the form  $(x - a)^2 + (y - b)^2 = r^2$  where  $(a, b)$  are the coordinates of the centre of the circle and  $r$  is the radius. This form of the circle equation was most suitable for the solution required. Some candidates correctly recalled the expanded form  $x^2 + y^2 - 2fx - 2gy + c = 0$ .

In Part (b) (i), candidates recognised that the dot product  $\overrightarrow{RT} \cdot \overrightarrow{RS} = 0$  was the condition required for proving that  $\angle TRS = 90^\circ$ . Other conditions presented in candidates' responses were for angle  $\angle RTS = 90^\circ$ ,  $\cos \angle RTS = \frac{\overrightarrow{RT} \cdot \overrightarrow{RS}}{|\overrightarrow{RT}| |\overrightarrow{RS}|} = 0$ , requiring knowledge of the length of a vector.

$$|\overrightarrow{RT}| \cdot |\overrightarrow{RS}|$$

- The product gradient  $\text{RT} \times \text{gradient RS} = -1$
- Graphical method
- An unknown method involving ratios

In Part (b) (ii) candidates correctly recalled Pythagoras' Theorem to determine the length of the hypotenuse. Candidates faced challenges throughout their responses in terms of correctly applying the content to data.

In Part (a) (i) candidates were unable to manipulate the two equations, given in the form  $-x + 3y = 6$  and  $kx + 2y = 12$ , to obtain the gradients. Only a few candidates correctly identified  $-k/2$  as one of the gradients. As a consequence candidates were unable to formulate the equation  $-1/3 \times -k/2 = -1$ , resulting in incorrect values for  $k$ .

The layout of both the given equations,  $x + 3y = 6$  and  $kx + 2y = 12$  prompted candidates to obtain the value of  $k$  by solving the equations simultaneously, not recognising that two equations containing three unknowns cannot be solved. This attempt by candidates was abandoned at some point in the procedure.

The follow through allowed in Part (a) (ii) for the incorrect value of  $k$  made it possible for candidates to be awarded both marks for solving the simultaneous equations to obtain the coordinates of the centre of the

circle. Some candidates wrote the equation of the circle incorrectly due to the non-reversal of signs when substituting the coordinates of the centre of the circle in the correct equation of the circle.

The use of the word “show” in the stem of Part (b) (i) led to the use of a variety of strategies in the proof.

In both Parts (b) (i) and (b) (ii) incorrect selection of vectors for substitution in the dot product or the formula for the length of a straight line was frequently seen. Position vectors were commonly seen rather than the correct translation vectors. Incorrect terminology in writing vectors was also seen e.g.  $\vec{RT} \bullet \vec{RS}$  was written  $R \times S$ ).

Consolidation in Coordinate Geometry is required in the area of expressing equations of straight lines given the form  $ax + by = c$  to the form  $y = mx + c$  in order to better facilitate the identification of the gradient. Competency in developing proofs using a vector method also needs to be emphasized.

- Solutions are:**
- (a) (i)  $k = -6$   
(ii) Equation of the circle  $(x + \frac{6}{5})^2 + (y - \frac{12}{5})^2 = 5^2$
- (b) (i)  $\vec{RT} \cdot \vec{RS} = 0$  for angle  $\text{TRS} = 90^\circ$   
(ii)  $|\vec{ST}| = \sqrt{26}$

#### Question 4

This question tested candidates' ability to:

- determine the area of a sector of a circle and find the area of a shaded region using the area of the sector and that of a triangle;
- use the compound angle formula for  $\cos(x + \frac{\pi}{6})$
- prove a trigonometric identity.

Candidates performed best on Part (b), which required proving the compound angle formula. Candidates were able to apply the formula given in the formula sheet and correctly substitute the values given. Candidates were also successful in factorizing the  $\frac{1}{2}$  to give the result. Some candidates, though, used both sides of the proof to attain the answer.

Teachers are urged to show the students the method of substituting correctly and how to use either the left hand side or the right hand side of the equation to achieve the proof.

Part (a) presented the most difficulty for candidates. A number of candidates incorrectly calculated the area of the triangle and subsequently calculated an incorrect area of the shaded region,  $H$ . However, most candidates knew that the area of the shaded region,  $H$ , was calculated by subtracting the area of the sector AOB from the area of the triangle OAC. Most candidates also attained full marks for Part (i) in finding the area of the sector. Teachers are encouraged to have students understand the steps needed to complete questions of this nature, that is, the correct use of the formula  $\text{Area} = \frac{1}{2} ab \sin C$ .

In Part (c), most candidates received marks for substituting correctly for  $\tan \theta = \frac{\sin \theta}{\cos \theta}$  and  $\sin^2 \theta = 1 - \cos^2 \theta$ . However, simplification after the substitution, to get the required result  $1 + \frac{1}{\cos \theta}$ , proved to be most difficult. Candidates did not recognize the difference of squares  $1 - \cos^2 \theta = (1 + \cos \theta)(1 - \cos \theta)$  and, as a result, lost marks at that part of the proof.

Teachers are encouraged to reinforce the use of the difference of squares examples including  $1 - \cos^2\theta$  and  $1 - \sin^2\theta$ .

- Solutions are:**
- (a) (i) Area of sector =  $28.35 \text{ cm}^2$
- (ii) Area of shaded region =  $5.76 \text{ cm}^2$

### Section 3: Introductory Calculus

#### Question 5

This question tested candidates' ability to:

- determine the equations of the tangent to a curve;
- use the concept of stationary points; locate the maximum and minimum stationary points, by considering the sign changes of the derivative; calculate the second derivative of polynomials; and use the sign of the second derivative to determine the nature of stationary points.

Candidates' demonstration of the understanding of the Calculus and Coordinate Geometry content in this section of the syllabus was commendable. Candidates recognised that in Part (a)

- the first derivative would give rise to an expression for the gradient of the tangent,  $-2x + 4$ .
- substituting the  $x$ -coordinate of the given point in  $-2x + 4$  would give the value of gradient of the tangent.
- the general linear equation  $y = mx + c$  or  $y - y_1 = m(x - x_1)$  would give rise to the equation of the tangent of the curve.

For Part (b) (ii)

- differentiating the function  $f(x) = 2x^3 - 9x^2 - 24x + 7$  to obtain the first derivative  $f'(x) = 6x^2 - 18x - 24$  and equating the resulting quadratic equation to zero would lead to solutions which are the stationary points of the curve.
- further differentiating the first derivative to obtain the second derivative  $f''(x) = 12x - 18$  would enable them to determine the nature of EACH of the stationary points.

In Part (a) the candidates showed competence in correctly obtaining the first derivative  $dy/dx = 4 - 2x$  to represent the gradient of the given quadratic curve  $y = 3 + 4x - 3x^2$  and substituting  $x = 3$  from the given point to obtain  $m = -2$ .

In Part (b) (i) candidates solved the quadratic equation  $6x^2 - 18x - 24 = 0$  using the methods of the factorisation, completing the square and use of the quadratic formula.

In Part (b) (ii) candidates were able to obtain the second derivative  $f''(x) = 12x - 18$ . The follow through  $x$  values from Part (b) (i) was an added advantage to candidates who were able to substitute in  $f''(x) = 12x - 18$  to differentiate between the maximum and minimum stationary points of the function. Many candidates obtained full marks in this part of the question.

Other strategies used in candidates' responses to differentiate between the maximum and minimum stationary points were as follows:

- Sketching the graph of  $f(x)$
- Examining the coefficient of  $x^3$  to determine the shape of the graph and the direction of the stationary points.

Areas of ambiguity in Part (a) were

- whether the equation of the tangent to the curve or the normal was required. Some candidates converted the gradient  $m = -2$  to  $m = \frac{1}{2}$  in order to substitute into general equation thus finding the equation of the normal to the curve at P (3, 6) instead of the equation of the tangent.
- the use of the given point on the curve P (3, 6). Some candidates used the gradient expression to formulate the equation  $-2x + 4 = 0$  to find a new value of  $x = 2$  for substituting in the gradient expression. Others used the given function to formulate a quadratic equation in order to find  $x$  values for substituting in the gradient expression.
- multiplying through the curve  $y = 3 + 4x - x^2$  by  $-1$  before differentiating. This gave rise to the incorrect gradient expression  $2x + 4$ .

Weakness in basic algebraic manipulations was evident in this part of the question, including

- transposition of terms in an equation
- factorisation of quadratic expressions.

Transposing the equation of the tangent obtained to the required format posed a challenge to candidates who often expressed their equation in the form  $y = mx + c$ . Attempts at transposing the equation showed incorrect signs associated with the coefficients of  $x$  and  $y$ .

In Part (b) (i), factorising the quadratic equation  $6x^2 - 18x - 24 = 0$  was a challenge to many candidates. Incorrect factors were obtained. Use of the quadratic formula and completion of the square method were often unsuccessful and abandoned before arriving at solutions to the equation.

Some candidates who obtained the correct  $x$ -coordinates did not proceed to calculate the corresponding  $y$ -coordinates to obtain ALL the stationary points.

With respect to candidates' responses in this question, there was good demonstration of understanding of the content areas throughout the question. Deficiencies in algebraic manipulations for weaker candidates can be a drawback to these candidates' performance. These deficiencies can be eliminated by consistent and appropriate worksheets with specific focus on developing skills in algebraic manipulations with accompanying effective feedback to candidates.

- Solutions are:**
- (a) Equation of the tangent to the curve at (3, 6) is  $2x + y - 12 = 0$ .
  - (b) (i) Stationary points are  $(-1, 20)$  and  $(4, -105)$
  - (ii)  $(-1, 20)$  maximum point,  $(4, -105)$  minimum

### Question 6

This question tested candidates' ability to:

- evaluate a definite integral;
- compute a definite trigonometric integral leaving the answer in surd;
- formulate the equation of a curve given its gradient function and points on the curve; find the area of a finite region in the first quadrant bounded by a curve and the  $x$ -axis and the lines  $x = 3$  and  $x = 4$ .

Candidates performed best on Part (b), except for those candidates that did not know the surd form for  $\cos \frac{\pi}{3}$  and  $\sin \frac{\pi}{3}$ . Part (c) (i) was fairly well done; however, there were many candidates who were unable to correctly calculate the value of the constant of integration,  $c$ , despite the fact that they were able to correctly integrate the function and substituted the point  $(2, -5)$ . In Part (c) (i), some candidates could not



obtain the correct equation of the curve. Even among those who knew to integrate the gradient function to obtain this equation, some did not include a constant of integration and hence obtained an incorrect equation.

Part (a) and Part (c) (ii) presented the most difficulty to candidates. In Part (a), many candidates used the incorrect formula. “the integral of a product is the product of the integrals”. Others did not integrate at all, but simply substituted the limits in the function.

In Part (c) (ii), generally candidates knew that area is the integral of a function between limits. However, many used the function as  $y = 6x^2 - 1$ , which was actually the given gradient function,  $\frac{dy}{dx} = 6x^2 - 1$ , instead of the equation of the curve obtained in Part (c) (i).

- Solutions are:**
- (a) 48
  - (b)  $2\sqrt{3} + 1$
  - (c) (i)  $2x^3 - x - 19$  Area = 65 units<sup>2</sup>

#### Section 4: Basic Mathematical Applications

##### Question 7

This question tested candidates' ability to:

- use the formula  $P(A \cup B) = P(A) + P(B) - P(A \cap B)$  to calculate probabilities;
- construct and use possibility space diagrams to solve problems involving probability;
- use the given stem and leaf diagram to determine quartiles and the median and to construct a box- and-whisker plot.

Candidates performed best on Part (b) (i), which required them to complete the possibility space diagram, and they also able to interpret the correct sample space of 36. For Part (a), most candidates used previous knowledge of sets and Venn diagrams to evaluate the correct probabilities.

Candidates performed well on the interpretation of the stem and leaf diagram to calculate the median and quartiles for the data given, recognizing that the median was the 26<sup>th</sup> observation, 71. The majority of the candidates also knew to construct a box-and-whisker plot to illustrate the data given. However, the comment on the skewness of the distribution proved to be very difficult. Candidates opted to omit the answer of negatively skewed and some did not provide a mathematical comment on the plot.

Teachers are advised to stress on the skewness of a box-and-whisker plot to describe the distribution of the data. The box-and-whisker plot should be accurately drawn on graph paper as points were deducted for incorrect scale.

Part (b) (iii) was also difficult in that most candidates who attempted to calculate the difference between the two dice forgot to multiply by 2 since the answers could be reversed. Very few candidates used a possibility space diagram to calculate the probabilities, but instead listed the possible combinations.

Generally, candidates displayed knowledge on the Statistics option but made some simple mistakes. For example, candidates need to know that probabilities are between 0 and 1 inclusive and that any answer above 1 is incorrect.

Candidates performed best on Part (a) (i) of this question, which required them to complete the construction of a tree diagram. Candidates knew that the sum of each branch equated to 1. However, having drawn the tree diagram, many candidates did not use it to answer Parts (a) (ii) and (iii). Part (a) (ii), on the conditional

probability, was badly done, and it appeared that most candidates did not even know that the question required the use of conditional probability. In Part (a) (iii), most candidates were able to get the probability for Event V, and also to say which was the more likely event.

Parts (b) (i), (ii), (iv) and (v) presented the most difficulty to candidates. In Part (b) (i), which asked candidates to state an advantage of a stem and leaf diagram versus a box and whiskers plot to display the given data, a number of candidates stated their personal preference rather than an advantage, for example, 'the stem and leaf is easier to draw and understand'. In Part (b) (ii), which required candidates to construct the actual stem and leaf diagram, a number of candidates drew actual tree diagrams. For candidates who did draw a stem and leaf diagram, a number of them lost a mark for not including a key to the diagram. In Part (b) (iv) candidates did not know the formula for semi-interquartile range, or did not know how to work this out. In many cases they could not correctly find the lower and upper quartiles, and many did not know to divide this difference by two. In Part (b) (v), some candidates mistook the sample space for the marks. Many other candidates did not recognize the sample space being reduced, and so could not correctly give the probability for the second student scoring less than 50 marks on the exam.

**Solutions are:** (a) (i)  $\frac{3}{20}$

(ii)  $\frac{11}{60}$

(b) (i)

<b>6</b>	7	8	9	10	11	12
<b>5</b>	6	7	8	9	10	11
<b>4</b>	5	6	7	8	9	10
<b>3</b>	4	5	6	7	8	9
<b>2</b>	3	4	5	6	7	8
<b>1</b>	2	3	4	5	6	7
	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>	<b>6</b>

(ii) a)  $P(S > 9) = \frac{1}{6}$

b)  $P(S \leq 4) = \frac{1}{6}$

(iii)  $a = \frac{5}{18}$        $b = \frac{1}{6}$        $c = \frac{1}{18}$

(c) (i) median = 71      lower quartile = 56      upper quartile = 79

(ii) Draw box-and-whisker plot  
Shape is negatively skewed

### **Question 8**

This question tested candidates' ability to:

- interpret and make use of velocity-time graphs to find distance, deceleration and time;
- apply the appropriate rate of change such as  $a = \frac{dv}{dt}$  and use such to calculate time when velocity is maximum, maximum velocity and distance moved by particle from rest to where the particle attains maximum velocity.

Candidates performed best on Parts (a) (i) and (iii) and (b) (i) and (ii) of this question which required them to calculate the distance from a velocity-time graph, the time it took to decelerate to  $10\text{ms}^{-2}$ , obtain the acceleration when the velocity is maximum and calculate the maximum velocity.

Parts (a) (ii), (iv) and (b) (iii), however, presented the most difficulty to candidates. In Part (a) (ii), candidates were required to find the time travelled by the particle when it started to decelerate, as well as to find the average acceleration of the particle. Most candidates looked at the graph and estimated the time as 10 seconds instead of using the formula distance = total area under the curve to calculate the correct time. A few candidates were able to correctly estimate that the car took 8 seconds to decelerate and then found the average velocity using average velocity =  $\frac{\text{distance}}{\text{time}}$ .

In Part (a) (iv), many candidates were unable to correctly find the additional time after 90 seconds for the particle to come to its final rest. Candidates did not use the area of the triangle to calculate the time, but rather some divided the 30 metres it further travelled by  $10\text{ms}^{-1}$ .

Part (b) (iii), required candidates to calculate the distance moved by the particle from rest to where the particle attained maximum velocity, that is they were expected to evaluate  $\int_0^5 0.72t^2 - 0.096t^3 dt$ .

Some candidates used the first function with limits between 5 and 10. Others used the second function with limits between 0 and 5. Still others integrated the first function with limits between 0 and "his" time that represented where the particle reached maximum velocity.

- Solutions are:**
- |     |       |  |
|-----|-------|--|
| (a) | (i)   | 750 m                                  |
|     | (ii)  | deceleration = $1.875\text{ m s}^{-2}$ |
|     | (iii) | 52 seconds                             |
|     | (iv)  | $15\text{ m s}^{-1}$                   |
| (b) | (i)   | $t = 5$ seconds                        |
|     | (ii)  | max velocity = $6\text{ m s}^{-1}$     |
|     | (iii) | distance travelled = 15 m              |

**Paper 03/2-Alternate to the School Based Assessment (SBA)**

This paper tested candidate ability to:

- construct an geometric series from circumscribed rectangles and obtain the sum of its first  $n$  terms.
- apply logarithms to a problem involving the transformation of a given relationship to a linear form and use its graph to unknown constants.

For Part (a), candidates were required to use the method of circumscribed rectangles to:

- determine the width of each of  $n$  subdivisions
- determine the point of each subdivision
- determine the height of each circumscribed rectangle
- determine area of each circumscribed rectangle
- determine the area of the circumscribed rectangles
- compute the area of the circumscribed rectangles as the progression increases
- determine the limit to which these values approaches

For Part (b), given a logarithm function:

- convert to the linear form
- find the values  $a$  and  $n$  by finding gradient and  $y$  intercept

There were approximately 120 responses to this paper. No candidate received the full 20 marks. The highest mark attained was 17. The mean score was 8.69 with a standard deviation of 7.75

Candidates, in general, were unable to do Part (a) of the question. About 50 % substituted into  $S_n$  to get values for  $a$  but were unable to identify that the limit approached 0.5.

For Part (b) most candidates were successful in finding the logarithms of the table and plotting the graph. The scale did not accommodate all the points to be plotted. Therefore, some of the candidates were unable to make the necessary adjustments. They were however able to convert the equation to the logarithm format, as well as to calculate the values of  $a$  and  $n$  from the graph. In general, the question was poorly done.

**Solutions are:**

- (a) (i)  $\frac{1}{n}$  .
- (ii)  $x_1 = \frac{1}{n}$  ,  $x_2 = \frac{2}{n}$  ,  $x_3 = \frac{3}{n}$  .....  $x_{n-1} = \frac{n-1}{n}$  .
- (iii)  $h_1 = \frac{1}{n}$  ,  $h_2 = \frac{2}{n}$  ,  $h_3 = \frac{3}{n}$  , .....  $h_n = 1$
- (iv)  $A_1 = \frac{1}{n^2}$  ,  $A_2 = \frac{2}{n^2}$  ,  $A_3 = \frac{3}{n^2}$  .....  $A_n = \frac{1}{n}$  .
- (vi) 0.55 0.525 0.51 0.505
- (b) Approaches 0.5 as  $n$  gets larger
- $\log y = \log a + n \log x$   
 $n$  is approximately 4  
 $a$  is approximately 3 .

### **Paper 031 – School-Based Assessment (SBA)**

Many of the projects submitted were of a high quality and generally related to real-world situations. The mean mark awarded on the SBA was 33.06 (out of a possible score of 40 marks), with standard deviation of 6.30.

A number of observed weak areas related to sample submissions, related to the submissions themselves as well as to the marking of these samples. The following were general observations on both Project A and Project B:

- Some SBA project titles and aim(s) or purpose were not clear, not concise and not well defined. Some titles were too vague, others too wordy.
- In far too many cases, students simply re-stated the project's title as its aim or purpose. This often meant that the reason for the project was not stated.
- In a number of cases, the project's aim or purpose did not relate to the given title and sometimes was not linked to any syllabus objective;
- Some submissions did not come with the required CSEC Additional Mathematics rubric;
- Some teachers created their own rubric or made adjustments to the CSEC Additional Mathematics rubric. This is unacceptable, as the project then has to be re-marked using the appropriate CSEC Additional Mathematics rubric;
- A few project submissions appeared to have been downloaded from the Internet;
- There were few instances of incorrect spelling and grammar. However, candidates used the correct mathematical jargon and symbols appropriately most of the time;
- Some projects were incorrectly categorized and assessed, that is, a Project A being labelled an assessed as a Project B, and vice versa. This did create some problems as, for example, Project B requires the collection and analysis of data from an experimental-type activity.

#### **Comments specifically for Project A:**

##### **Mathematical Formulation**

- In stating how the project was going to be done most students correctly identified all the important elements of the problem and showed understanding of the relationship between the elements.

##### **The Problem Solution**

- Assumptions were often not clearly stated. Some students, though, did state limitations of the project.
- Explanations were generally not sufficient and also were not placed between every successive step in the problem solution.
- Calculations were often precise but solutions were not clearly stated.

##### **Application of Solution**

- Many students were unaware of how to show that the solution or proof of the given problem was valid; was necessary to see how the candidate's solution could be applied elsewhere or to be able to substitute their values to check for validity.

##### **Discussion of Findings/Conclusion**

- In some instances the discussion was worthwhile. However, the discussion was not always related to the project's purpose and was often quite broad.
- Conclusions in many instances were not found to be valid. Some candidates stated many conclusions which were not related to the purpose of the project.
- There were few instances where suggestions for analyses were not stated or included in the project.

## **Comments specifically for Project B:**

### **Method of Data Collection**

- Although students indicated what they did, some were not specific enough about the type of data collected and method of sampling. Since the marking rubric was not specific as to what is needed, the candidates were not penalized.

### **Presentation of Data**

- Too many students had the minimum required one table or chart; however, some tables and charts were not properly named or labelled, for example, graphs produced using software programs.
- There was not a systemic layout or organization of the tables and charts

### **Mathematical Knowledge (Analysis of Data)**

- In this section, the student was expected to outline HOW the project was going to be done.
- Mathematical concepts in many cases were not used appropriately from the Additional Mathematics syllabus. Many projects utilized content at Form 3 level or below.
- Most of the calculations shown were accurate. However, in some cases no working was seen because some students used software to generate the values.
- Generally some analysis was attempted, but the analysis was often not coherent. This could be attributed to the fact that no proper summary table of calculation values was seen, so the markers had to search through the document constantly to link the calculated values with the analysis.
- The students were often not very clear on which two approaches were to be used in the analysis. In many cases the two approaches used were of the same concepts, for example, mean, mode and median were presented as different approaches to the analysis, but all of these are measures of central tendency.

### **Discussion of Findings/Conclusions**

- In most instances there was no statement of findings, in some cases if it was in the project, it was not properly placed.
- Conclusions made were based on the reported findings but often were not related to the purpose of the project. As a result this made their conclusions invalid.
- In few cases there were no suggestions for future analysis. This was an improvement for last year 2013.

### **Plagiarism**

Plagiarism continued to be a problem in the Additional Mathematics projects bring submitted for moderation. There were cases where projects were taken directly from the specimen papers as well from the exemplar projects given in the Additional Mathematics syllabus. This was noted among last year's submissions, and the practice continued in this year's submissions. The concern here is that the teachers associated appeared to be accepting these as legitimate projects, when it is clear that they are not the candidates' own work. This is must be stopped.

There were other instances where candidates had obviously done the project as a group. However, they submitted the same data, same mathematical formulations/solution, same diagrams, graphs and tables, same discussions and same conclusion, in some cases even printing and submitting identical copies. This practice must be stopped. Even where the project is done as a group, ALL aspects of the write-up must show candidates' individual effort. Only the raw data collected must be the same.

The following recommendations hold for continued improvement in this aspect of the Additional Mathematics examinations:

- All projects should have a clear and concise title, and well-defined aim(s) or purpose;
- Where possible the SBA should be relevant to authentic situations;

- The variables that are being used (Project B) must be clearly stated and described. Variable can be controlled, manipulated and responding;
- The type of sample and sample size, if relevant, must be clearly stated;
- Teachers must ensure that projects which integrate other subject areas utilize concepts as contained in the CSEC Additional Mathematics syllabus;
- If students collect their data in a group setting, they MUST demonstrate their INDIVIDUAL effort in relation to analysis (interpretation) and finding(s)/conclusion(s);
- Projects involving dice or playing cards must be more expansive so that candidates can present a more in-depth analysis of the topic under consideration;
- As good practice, students should be encouraged to cite all sources and insert a reference/bibliography page;
- Teachers should guide students using the assessment criteria found in forms ‘Add Math 1- 5A’ and ‘Add Math 1-5B’ which are both available on the CXC website. Teachers can give their students the rubric as a means of guidance in developing their projects.

It is clear that training workshops are needed in the area of project work for Additional Mathematics. Therefore, it is recommended that workshops be held throughout the Caribbean. Teachers need to be aware that there are TWO RUBRIC SHEETS 1-5A for Project A (the mathematical modelling project) and 1-5B for Project B (the statistical project), and to use the appropriate sheet in marking students’ projects. It was apparent that some teachers found difficulties in understanding some aspects of the rubric, or were unaware of the existence of the rubric. Teachers need to ensure that the content level in the project coincides with the Additional Mathematics syllabus.

### **Paper 032 – Alternative to School-Based Assessment (SBA)**

This paper tested candidates’ ability to:

- construct a geometric series from circumscribed rectangles and obtain the sum of its first  $n$  terms.
- apply logarithms to a problem involving the transformation of a given relationship to a linear form and use its graph to unknown constants.

For Part (a), candidates were required to use the method of circumscribed rectangles to:

- determine the width of each of  $n$  subdivisions
- determine the point of each subdivision
- determine the height of each circumscribed rectangle
- determine the area of each circumscribed rectangle
- determine the area of the circumscribed rectangles
- compute the area of the circumscribed rectangles as the progression increases
- determine the limit to which these values approaches

For Part (b), given a logarithm function:

- convert to the linear form
- find the values  $a$  and  $n$  by finding gradient and  $y$  intercept

There were approximately 120 responses to this paper. No candidate received the full 20 marks. The highest mark attained was 17. The mean score was 8.69 with a standard deviation of 7.75

Candidates, in general, were unable to do Part (a) of the question. About 50 per cent substituted into  $S_n$  to get values for  $a$  but were unable to identify that the limit approached 0.5.

For Part (b) most candidates were successful in finding the logarithms of the table and plotting the graph. The scale did not accommodate all the points to be plotted. Therefore, some of the candidates were unable to

make the necessary adjustments. They were however able to convert the equation to the logarithm format, as well as to calculate the values of  $a$  and  $n$  from the graph. In general, the question was poorly done.

- Solutions are:**
- (a) (i)  $\frac{1}{n}$  .
- (ii)  $x_1 = \frac{1}{n}$  ,  $x_2 = \frac{2}{n}$  ,  $x_3 = \frac{3}{n}$  .....  $x_{n-1} = \frac{n-1}{n}$  .
- (iii)  $h_1 = \frac{1}{n}$  ,  $h_2 = \frac{2}{n}$  ,  $h_3 = \frac{3}{n}$  ,.....  $h_n = 1$
- (iv)  $A_1 = \frac{1}{n^2}$  ,  $A_2 = \frac{2}{n^2}$  ,  $A_3 = \frac{3}{n^2}$  .....  $A_n = \frac{1}{n}$  .
- (vi) 0.55 0.525 0.51 0.505
- (b) Approaches 0.5 as  $n$  gets larger
- $\log y = \log a + n \log x$   
 $n$  is approximately 4  
 $a$  is approximately 3 .



**C A R I B B E A N      E X A M I N A T I O N S      C O U N C I L**

**REPORT ON CANDIDATES' WORK IN THE  
CARIBBEAN SECONDARY EDUCATION CERTIFICATE® EXAMINATION**

**MAY/JUNE 2015**

**ADDITIONAL MATHEMATICS  
GENERAL PROFICIENCY EXAMINATION**

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## GENERAL COMMENTS

Additional Mathematics was tested for the fourth time in the May/June 2015 CSEC examinations. The subject is intended to bridge a gap between CSEC General Proficiency Mathematics and the CAPE Unit 1 Pure Mathematics.

The intention is to offer candidates progressing from the CSEC level in Mathematics to the CAPE Unit 1 level in Pure Mathematics/Applied Mathematics a seamless transition to the advanced thinking required for these courses, although a candidate who masters the content set out in the CSEC syllabus for General Proficiency Mathematics should be armed with the tools needed to be successful at the CAPE proficiency level.

As established, the examination consists of three papers:

- Paper 01 – a 45-item multiple choice paper
- Paper 02 – a structured, ‘essay-type’ paper consisting of eight questions
- Paper 031 – a School-Based Assessment (SBA) project component for candidates in approved educational institutions
- Paper 032 – an alternative to SBA for out-of-school (private) candidates

Questions in the examination targeted the content and specific objectives of the Additional Mathematics syllabus (CXC 37/G/SYLL 10) with the aim of ensuring a wide coverage of the syllabus. The questions were designed at the appropriate level to test the following skills: Conceptual Knowledge, Algorithmic Knowledge and Reasoning.

Paper 01 tests content from Sections 1, 2 and 3 of the syllabus and consists of 45 items. Paper 02 tests content from all four sections of the syllabus. This year the paper consisted of four sections and each section contained two problem-solving questions. The questions in Section 1, 2 and 3 were all compulsory and each question in these sections were worth 14, 12 and 14 marks respectively. Section 4 contained two questions, one on Data Representation and Probability and the other on Kinematics and each question was worth 20 marks. Candidates were required to answer only one question from this section. Paper 031 is the SBA component of the examination. Candidates are required to do one project chosen from two project types: a mathematical modelling project (Project A) and a data handling/statistical analysis project (Project B). The SBA component is worth 20 marks. Private candidates can sit an alternate paper to the SBA, Paper 032, which consists of one in-depth, extended question from Sections 1, 2 and/or 3 of the syllabus. This paper is worth 20 marks.

The percentage of candidates earning Grades I–III was 73 per cent, compared with 77 per cent in 2014.

This year saw an 11.6 per cent increase in candidates registered for the examination, up from 3609 candidates in 2014 to 4016 in 2015.

Again this year, a formula sheet was included as part of the question paper on Papers 01, 02 and 032.

## DETAILED COMMENTS

### Paper 01 – Multiple Choice

This was a 45 item paper covering Sections 1, 2 and 3 of the syllabus. The mean score on this paper was 39.54 with a standard deviation of 13.36 respectively, compared with 39.41 and 12.71 in 2014.

### Paper 02 – Structured Essay Questions

This paper consisted of eight questions, of which Questions 1 to 6 were compulsory. Candidates had to choose one of Questions 7 and 8. The total possible score on this paper was 100 marks. The mean and standard deviation for this paper were 47.42 and 26.98 respectively, compared with 46.32 and 27.27 in 2014.

### Section 1: Algebra and Functions

#### Question 1

This question tested candidates' ability to:

- Determine the value of  $g(f(2))$  given the two functions  $g$  and  $f$ .
- Determine the inverse of a function given in the form  $\frac{m(x)}{n(x)}$ , where  $m(x)$  and  $n(x)$  are linear functions.
- Completely factorize a cubic equation given one of its linear factors.
- Use the laws of indices to solve an indicial equation.  
Use the laws of logarithms to solve a logarithmic equation.

There were about 4016 responses to this question. The mean mark was 8.41 with a standard deviation of 4.19.

Candidates performed best on Parts (a) and (b), as they should at this level. Candidates who did poorly in these parts generally performed poorly throughout the entire question.

In Part (a), most candidates were able to achieve full marks. Some candidates attempted to substitute  $f(x)$  into  $g(x)$  and made mistakes while doing the algebra. Few candidates were seen calculating  $f(2)$  first and then substituting this value into  $g(x)$  which resulted in fewer mistakes.

#### Common Errors

Simple arithmetic miscalculations were made, for example,  $20 - 3$  was given as  $-17$  instead of  $17$ .

The majority of candidates performed very well on Part (b). Even weaker candidates were able to achieve a mark by interchanging  $x$  and  $y$ .

#### Common Errors

Candidates misread the original function, which led to incorrect factorization of  $y$ .

Candidates 'forgot' to interchange  $x$  and  $y$ .

Part (c) was done relatively well. The majority of candidates recognized the application of the factor theorem and correctly used the long division method to obtain a quadratic equation, which was then factorized. Some used a comparison of coefficients approach and a trial and error approach while a few used the synthetic division method in answering this question. The candidates who were most successful used the long division method.

### Common Errors

Candidates did not factorize their quadratic quotient to obtain the other two linear factors.

Many proved  $(x - 2)$  was a factor of  $k(x)$ , but this was given in the question.

It appears as though some candidates used programmable calculators to find the roots of the cubic equation and were therefore unable to provide the necessary supporting statements. Simple algebraic/arithmetic errors were seen in the long division method.

In Part (d) (i), candidates recognized the need to change each side to a common base, but many of them were unable to correctly apply laws of indices and solve the resulting simple linear equation presented by the equating of indices. Some candidates applied logarithms to both sides, but few were successful as they applied the laws of logarithms incorrectly.

### Common Errors

Equating indices of different bases

Expanding brackets incorrectly when simplifying the indices

Inability to recognize  $\frac{1}{4} = 4^{-1}$  (negative indices)

Part (d) (ii) presented the most difficulty to candidates. Candidates did not demonstrate a sound knowledge in logarithms. Most showed confusion when ‘dropping logs’, as they simply removed the log function and proceeded to solve the question as is.

Some candidates who correctly applied the laws of logarithms, transposed incorrectly resulting in an incorrect quadratic equation while those who obtained the correct quadratic equation lost a mark as a result of their inability to factorize correctly.

### Common Errors

$\log_a M + \log_a N = \log_a (M + N)$  instead of  $\log_a M + \log_a N = \log_a (MN)$

$3^0 = 0$  instead of  $3^0 = 1$

### Factorization Errors

With respect to candidates’ responses on this question, it is recommended that candidates be given more practice questions. This will allow them to understand and apply appropriate laws where and when needed.

### Solutions

(a)  $gf(2) = 33$

(b)  $h^{-1}(x) = \frac{2x+5}{x-3} \quad x \in \mathbf{R}, x \neq 3$

(c)  $2x^3 - 5x^2 + x + 2 = (x-2)(x-1)(2x+1)$

(d) (i)  $x = -2.5$  (ii)  $x = 2$  and  $x = 3$

## Question 2

This question tested candidates' ability to:

- Express a quadratic function  $f(x)$  in the form  $a(x+b)^2 + c$  where  $a$ ,  $b$  and  $c$  are constant
- Determine the coordinates of the function's minimum point.
- Use the sum and product of the roots of a quadratic equation to determine the value of  $\frac{1}{\alpha} + \frac{1}{\beta}$ , where  $\alpha$  and  $\beta$  are the roots of the equation.
- Determine, algebraically, the point of intersection of a non-linear curve and a straight line.
- Use the formula for the  $n^{\text{th}}$  term of an arithmetic progression to calculate a particular term given the first term and the common difference.

There were 3522 responses to this question. The mean mark was 8.79 with a standard deviation of 4.18.

The overall performance on this question was good. About 55 per cent of the candidates were able to score full marks.

In Part (a), the majority of candidates attempted to complete the square but only about 60 per cent were able to do so successfully. The majority of the candidates used the classical method to complete the squares and about 95 per cent of them were able to identify that  $a = 3$ , however, finding correct values for  $b$  and  $c$  proved to be difficult due to numerical errors they made along the way. Some candidates used other techniques to arrive at the values of their constants;  $3x^2 - 9x + 4 \equiv a(x+h)^2 +$

$k$ , where  $a = 3$ ,  $h = \frac{b}{2a} = -\frac{3}{2}$  and  $k = c - ah^2$  or  $\frac{4ac - b^2}{4a} = -\frac{11}{4} \rightarrow 3x^2 - 9x + 4 \equiv 3\left(x - \frac{3}{2}\right)^2 - \frac{11}{4}$  being the more common one. Some of the candidates using this approach did not

remember the formulae correctly;  $h = -\frac{b}{2a} = \frac{3}{2}$  and  $k = \frac{b^2 - 4ac}{4a} = \frac{11}{4}$  were some of the common mistakes made.

The majority of candidates was not able to provide the coordinates of the minimum point from their completed square form. They gave instead either the minimum value of the function or the  $x$  value at the minimum point only.

In Part (b), deriving the sum and the product of the roots from a given equation was well known but some candidates confused the signs in determining their answers;  $\alpha + \beta = \frac{b}{a}$  and  $\alpha\beta = \frac{-c}{a}$  were sometimes used. Other candidates used the numerical values they obtained by solving for  $\alpha$  and  $\beta$  instead of  $\alpha + \beta$  and  $\alpha\beta$ . It was also difficult for some candidates to express  $\frac{1}{\alpha} + \frac{1}{\beta}$  in terms of  $\alpha + \beta$  and  $\alpha\beta$ .

In Part (c), many candidates experienced great difficulty in simplifying the equation they obtained after eliminating one of the variables from the given equations. Some also did not proceed to find the coordinates of the points leaving only the  $x$  values they obtained. In many cases where they proceeded to find  $y$ , errors were often made.

In Part (d), candidates displayed a lack of knowledge about arithmetic progressions and geometric progressions when given in a real world context. Many attempted a trial and error approach rather

than using an arithmetic progression where  $T_n = a + (n-1)d$  and some even used a geometric progression. Some candidates used the first term as 38 400 instead of 36 000 while others used the formula for the sum of terms,  $S_n = \frac{n}{2}(2a + (n-1)d)$ .

Candidates need to be more vigilant when using equations and numerical figures given in the question paper in their solutions. Many of such errors were seen throughout the question.

### Solutions

(a) (i)  $3\left(x - \frac{3}{2}\right)^2 - \frac{11}{4}$  (ii) Minimum point is  $\left(\frac{3}{2}, -\frac{11}{4}\right)$

(b)  $\frac{1}{\alpha} + \frac{1}{\beta} = -\frac{3}{2}$

(c) Points of intersection are  $(-3, 37)$  and  $\left(-\frac{5}{2}, \frac{63}{2}\right)$

(d) Annual salary for the 9<sup>th</sup> year = \$55 200

## Section 2: Coordinate Geometry, Vectors and Trigonometry

### Question 3

This question tested candidates' ability to:

- Determine the coordinates of the centre of a circle and (ii) the length of its radius given the equation in the form  $x^2 + y^2 + 2fx + 2gy + c = 0$ ; (iii) determine the equation of the normal to a circle at a given point on the circle.
- Determine the unit vector in the direction of  $\overline{AB}$  given two vectors  $\overline{OA}$  and  $\overline{OB}$  written in the form  $ai + bj$ ; (ii) calculate the acute angle between the vectors  $\overline{OA}$  and  $\overline{OB}$ .

There were 2923 responses to this question. The mean mark was 6.03 with a standard deviation of 4.03.

Candidates' recall of knowledge of coordinate geometry and vectors was good.

In Part (a) (i), candidates were able to determine the coordinates of the centre of the circle by either completing the square or by using coordinates of centre  $= \left(\frac{-2f}{2}, \frac{-2g}{2}\right)$ , from the equation of the circle. The approach involving the coefficient method was the most popular one used by candidates.

In Part (a) (ii), finding the length of the radius was a challenge to most candidates. This was often the only mark lost in this section.

In Part (a) (iii), candidates were able to apply the gradient algorithm using the coordinates of the centre they obtained. However, some of them misinterpreted this gradient to be that of the gradient of the tangent and proceeded to obtain the gradient of the normal as the negative reciprocal of this result.

There was a unique solution of determining the **equation of the normal**:

$$(y_1 + g)x - (x_1 + f)y - gx_1 + fy_1 = 0$$

In Part (b) (i), candidates correctly obtained the vector  $\overrightarrow{AB}$ . Most candidates correctly calculated the modulus of  $\overrightarrow{AB}$  but were unable to write the **unit vector**  $\overrightarrow{AB}$

In Part (b) (ii), candidates were able to determine the dot product and the moduli of  $\overrightarrow{OA}$  and  $\overrightarrow{OB}$ . However, errors in computations led to an incorrect value of the required angle.

### Solutions

- (a) (i) coordinates of centre are (6, 11)  
 (ii) radius =  $\sqrt{5}$
- (b) (i) unit vector  $\overrightarrow{AB} = \frac{1}{\sqrt{29}}(2i - 5j)$   
 (ii)  $\hat{AOB} = 20.8^\circ$

### Question 4

This question tested candidates' ability to:

- Determine the area in a circle between a chord and an arc given the formula for the area of a sector.
- Use the trigonometric identity  $\sin^2 \theta = 1 - \cos^2 \theta$  to solve a quadratic equation with trigonometric arguments.
- Prove a trigonometric identity using the expansions for  $\sin 2\theta$  and  $\cos 2\theta$ .

Candidates performed best on Part (a), which required finding the area of the shaded region. Most candidates knew the formula for finding the required area. However, many candidates substituted  $30^\circ$  for the angle in the formula that involved radians.

In Part (b), many candidates knew that they had to substitute  $\sin^2 \theta = 1 - \cos^2 \theta$ . Most of the candidates who substituted correctly were able to simplify and form the quadratic equation. Many of them were able to solve the quadratic equation and obtain the required angles.

In Part (c), most candidates received marks for substituting correctly for  $\sin 2\theta = 2 \sin \theta \cos \theta$ . However, many did not realize that the substitution of  $\cos 2\theta = 2 \cos^2 \theta - 1$  was necessary to lead to the proof. Some of those who substituted correctly had difficulty in factorizing and simplifying and as a result were unable to complete the proof.

### Solutions

- (a) Area of shaded region =  $\frac{4}{3}\pi - 4 \text{ cm}^2$   
 (b)  $\theta = 104.5^\circ, 255.5^\circ$

### Section 3: Introductory Calculus

#### Question 5

This question tested candidates' ability to:

- Use the product rule to differentiate the product of a quadratic expression and a trigonometric function of the form  $\sin ax$  where  $a \in \mathbb{Z}$ .
- Determine the stationary points of a polynomial of order 3; determine the nature of the stationary points.
- Use the concept of connected rates to determine the rate of change of the radius of a spherical balloon w.r.t. time, given the rate of change of the volume w.r.t. time and the formula for the volume of a sphere.

There were approximately 2891 responses to this question. The mean mark was 6.35 with standard deviation of 4.06.

Candidates' understanding of the algorithmic procedures in the Introductory Calculus and Coordinate Geometry content in this section of the syllabus was generally good. However, most candidates were challenged by the real-life application of the rate of change in Part (c).

In Part (a), candidates easily obtained the derivative of  $2x^2 + 3$  to be  $4x$ . Their responses were often incorrect for  $\frac{d \sin 5x}{dx}$ . The coefficient 5 was often omitted and in a few instances  $-5$  or  $\frac{1}{5}$  were presented as coefficients of  $\cos 5x$ . In addition, many candidates left out the coefficient, 5, which was part of the argument of the sine function. Candidates correctly applied the product rule but the mark for simplifying their result was rarely obtained.

In Part (b), differentiating the function  $f(x) = x^3 - 5x^2 + 3x + 1$  to obtain the first derivative  $f'(x) = 3x^2 - 10x + 3$  and equating the resulting quadratic equation to zero to obtain the coordinates of the stationary points are successfully done. Candidates also correctly differentiated  $f'(x)$  to obtain the second derivative;  $f''(x) = 6x - 10$  which enabled them to correctly determine the nature of each of the stationary points. However, a few of them came to the wrong conclusions based on the signs of the second derivative; at the stationary points. Some of them concluded that a negative value of the second derivative at the stationary point resulted in a minimum point and a positive value resulted in a maximum point.

In Part (c), candidates did not show full understanding of the terminology for the derivatives. Assigning  $200 \text{ cm}^3 \text{ s}^{-1}$  correctly to  $\frac{dV}{dt}$  was not commonly seen and correct substitution into derivatives was not often done. In addition, formulating the chain rule was also challenging and hence the algorithmic mark for the correct result was often lost.

#### Solutions

- (a)  $(10x^2 + 15)\cos 5x + 4x \sin 5x$
- (b) (i) Coordinates stationary points are  $\left(\frac{1}{3}, \frac{40}{27}\right)$  and  $(3, -8)$
- (ii) Maximum point is  $(3, -8)$ ; Minimum point is  $\left(\frac{1}{3}, \frac{40}{27}\right)$



$$(c) \quad \frac{dr}{dt} = \frac{1}{2\pi}$$

### Question 6

This question tested candidates' ability to:

- Evaluate a definite integral for a trigonometric function with a multiple angle.
- Equate the derivative of a function to a given value to determine the value of an unknown constant.
- Formulate the equation of a curve given its gradient function and a point on the curve.
- Find the volume obtained by rotating a finite region, in the first quadrant bounded by a curve and the  $x$ -axis and the lines  $x = 0$  and  $x = 1$ , through  $360^\circ$ .

Part (a) was poorly done, very few of the candidates were able to integrate  $\cos 3\theta$  correctly. Those who remembered that the integral of  $\cos \theta$  is  $\sin \theta$  often gave the integral of  $\cos 3\theta$  as  $\sin 3\theta$ .

In Part (b) (i), approximately 65 per cent of the candidates were able to correctly calculate the value of the unknown constant,  $k$ , by substituting  $x = 2$  and equating the result to 14. A few of them substituted the value  $y = 3$  for  $\frac{dy}{dx}$ . This clearly demonstrates very superficial understanding of the topic.

However, the majority of the candidates was able to use their value of  $k$  to proceed to Part (b) (ii).

In Part (b) (ii), about 50 per cent of the candidates incorrectly used the equation of a line  $y = mx + c$ , as the equation of a curve. Candidates who correctly calculated the value of  $k$  sometimes had difficulty integrating correctly. In a few cases, the constant of integration was left out, which resulted in incorrect equations.

Part (c) was generally well done, with most of the candidates knowing the volume of revolution formula, as well as how to obtain the limits of integration. However, incorrect expansions of  $y^2$ , given  $y = x^2 + 1$  was frequently observed,  $x^4 + 1$  or  $x^4 + 2x^2 + 2$  were some of the expansions seen. Some candidates were very careless when integrating, they either forgot to integrate the constant term or integrated the variable terms incorrectly. Simple addition of fractions such as  $\frac{1}{5} + \frac{2}{3} + 1$  proved challenging for some candidates. Candidates also gave their answers in decimal form which reflected a lack of knowledge of exactness.

### **Solutions**

$$(a) \quad -\frac{1}{3}$$

$$(b) \quad (i) \quad k = 7$$

$$(ii) \quad \text{Equation of the curve is } y = \frac{7}{3}x^3 - \frac{7}{2}x^2 - \frac{5}{3}$$

$$(c) \quad \text{Volume generated} = \frac{28}{15}\pi \text{ units}^3$$

## Section 4: Basic Mathematical Applications

### Question 7

This question tested candidates' ability to:

- Use the concept of a possibility space to solve problems involving probability.
- Calculate the mean for grouped data.
- Construct a tree diagram and use it to calculate probabilities.

This question was the least popular of the optional questions with about 1530 candidates attempting it. It was poorly done with approximately 70 per cent of the candidates scoring under 10 marks, however, there were 29 candidates scoring full marks.

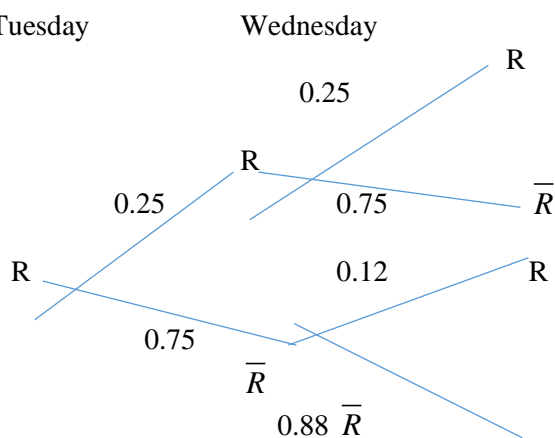
Candidates performed poorly on Part (a), which required finding the probability involving three independent events. Most candidates were able to identify that they were independent events. A few of the candidates worked with two events instead of the three events that were given because they were unable to correctly define the possibility space for the question.

Candidates performed best on Part (b), many of them knew that they had to calculate the midpoints in order to calculate the mean. However, far too many candidates could not do this part which is also covered in the CSEC General Mathematics syllabus.

In Part (c), the majority of the candidates received most of their marks from Part (c) (i), which required drawing a tree diagram. Some of the candidates did not label the branches properly; for example, they did not show their probabilities on the branches, while others did not label the days. Some of the candidates did not have the percentage sign (%) or the decimal equivalent for their probabilities. Most candidates knew that total probabilities on their branches should add to 1 and the majority of them attempted to use their tree diagram to calculate the probability of rain falling on Wednesday.

### Solutions

- (a) (i) Probability motorist has to stop at only one traffic light = 0.42  
(ii) Probability motorist has to stop at at least two traffic lights is = 0.5
- (b) Mean =  $\frac{340}{25} = 13.6$
- (c) (i)



Key: R means it rains;  $\bar{R}$  means it does not rain

- (ii) Probability it rain on Wednesday = 0.1525

Question 8

This question tested candidates' ability to:

- Draw and make use of a velocity–time graph for rectilinear motion to find acceleration and increase in displacement.
- Apply the result  $v = \int a dt$  and use it to calculate the velocity function, given the velocity at a specified time and use the result  $s = \int v dt$  to determine the displacement of a particle at a given instance of time.

There were approximately 2298 responses to this question. The mean mark was 10.27 with a standard deviation of 5.59. Two hundred sixty-three candidates obtained full marks (20 marks).

Candidates performed best on Parts (a) (i) and (ii). They were able to draw the correct velocity–time graph for the given information, as well as calculate the acceleration of the particle.

They used a variety of methods to determine the displacement in Part (a) (iii); some calculated the displacement by finding the area of a trapezium, others found the area of a rectangle plus the area of a triangle while others used equations of motion formulae or determined the equation of the velocity–time graph and integrated between the limits  $t = 0$  and  $t = 4$ . Approximately 25 per cent of the candidates only calculated part of the area under the graph.

The majority of the candidates who attempted Part (a) (ii), knew that the acceleration could be obtained using the formula  $\text{acceleration} = \frac{\text{change in velocity}}{\text{change in time}}$ . However, instead of using the initial and final velocities and the corresponding times, a few of the candidates used non- exact values which led to incorrect answers.

Parts (b) (i) and (ii) presented the most problems for more than half of the candidates. Many of them used equations of motion, which are only valid when the acceleration is constant, to do these parts. In Part (b),  $a = 3t - 1$  was given and candidates were required to obtain an expression for the velocity at the instant when the time was 4, and the displacement from the starting point at  $t = 3$ . Those who knew they had to integrate the velocity function to obtain the displacement sometimes forgot to include their constant of integration, which had to be calculated using the given information. This led to incorrect answers for both the velocity and displacement.

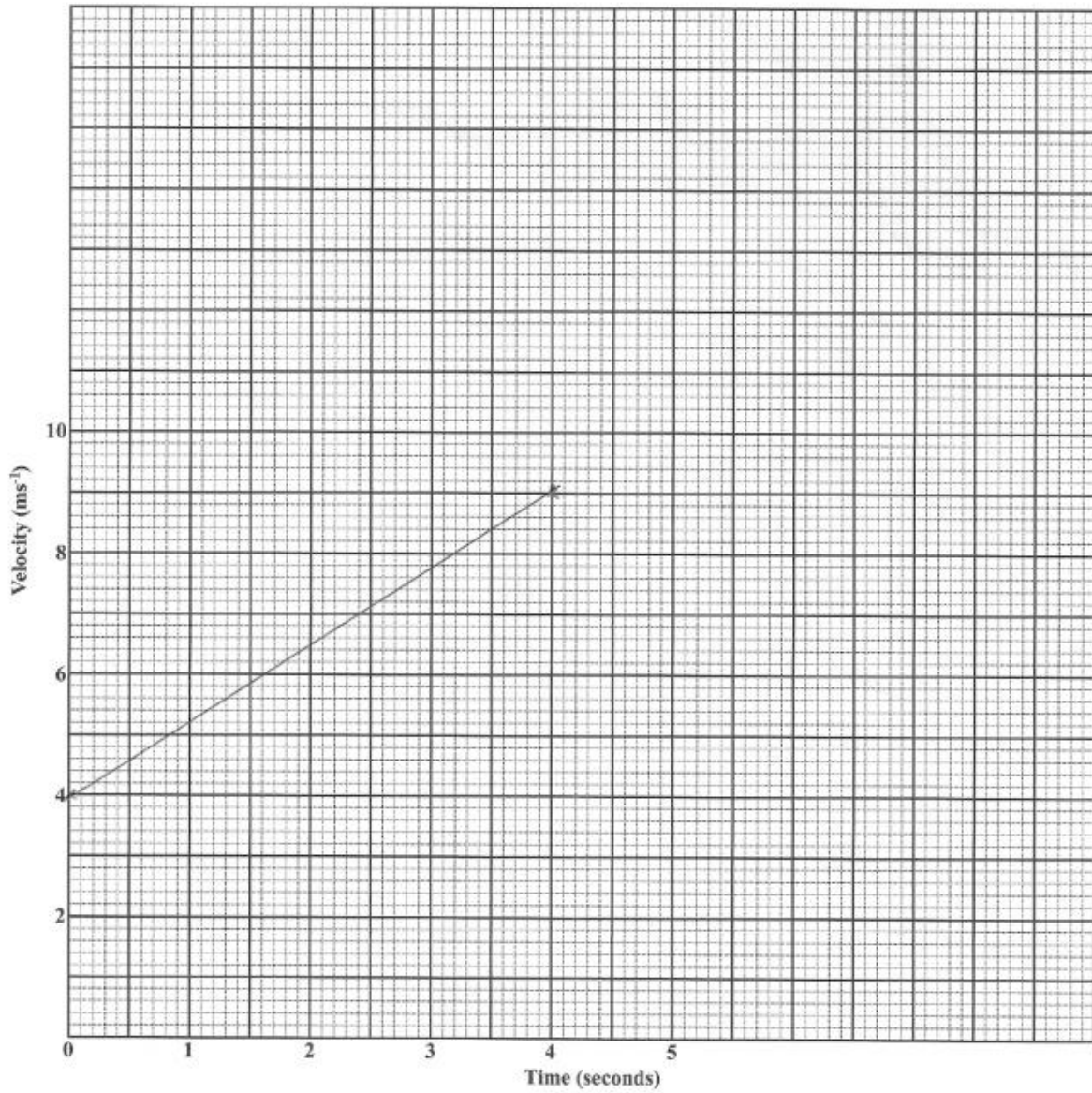
**Solutions****Recommendation for Teachers:**

- Students must be aware of the mathematical jargon associated with kinematics, for example, motion in a straight line (rectilinear motion) invariably means that the acceleration is constant.
- Students should be encouraged to use the information given in plotting a straight line graph to obtain the gradient, instead of choosing their own non-exact values.
- Where graph paper with the axes already written is provided, teachers should insist that that graph paper be used instead of any other graph paper.
- Students should be informed that the equations of motion (i)  $v = u + at$  (ii)  $s = ut + \frac{1}{2}at^2$  and (iii)  $v^2 = u^2 + 2as$  should only be used if the acceleration is constant.

- Teachers need to give a greater number of examples and practice questions after completing topics so that their students can get a clearer grasp of the concepts.

### Solutions

- (a) (i) See graph below  
(ii) Acceleration =  $1.5 \text{ m s}^{-2}$   
(iii) Displacement = 24 m
- (b) (i) Velocity =  $20 \text{ m s}^{-1}$  (ii) displacement = 13 m



### **Paper 031 — School Based Assessment (SBA)**

The marking team moderated many more projects this year from 191 examination centres, a further indication of the increase in the number of new schools as well as students pursuing studies in Additional Mathematics. Many of the projects submitted were of a high quality and generally related to real-world situations. The mean mark awarded on the SBAs was 32.08 (out of a possible total of 40 marks), with a standard deviation of 6.33, compared with 33.06 and 6.30 respectively in 2014. Many of the sample submissions received full marks.

#### **Comments specifically for Project A:**

##### **Title — *what is the project investigating?***

- Many of the students gave a name for the project rather than a title, these needs to be more specific.
- In most cases, students related to real world problems.

##### **Purpose of Project — *why this project/investigation is being done?***

- In many cases, the purpose was a replica of the title.
- Few students were specific enough about the purpose of the project.

##### **Mathematical Formulation — *how the problem described in the purpose is going to be investigated/solved***

- Generally, most students correctly identified all the important elements of the problem and showed understanding of the relationship between elements.
- In many cases, students did not use appropriate additional mathematical methods/models to solve the problems.
- The content used in some cases reflected Form 3/Grade 8 level mathematics, for example, some projects had only simple calculations with area and volume.

##### **The Problem Solution — *here the problem is solved***

- The assumptions were not clearly stated (must be explicit and not simply implied). However, some students stated limitations of the project.
- Explanations were not sufficient and not placed between every successive step in the problem solution.
- Calculations were precise but solutions were not clearly stated.

**Application of Solution** — *here the solution derived is applied to the real-world context*

- This section was fairly well done. However, many students were unaware of how to show that the solution or proof of the given problem was valid.
- Validation can be achieved by an alternative method or by comparison to a similar study.

**Discussion of Findings/Conclusion** — *interprets the solution and relates it to the purpose of the study*

- The discussions were too general and in most cases the findings/solutions were not related to the purpose of the project.
- The conclusions in many instances were found not to be valid. Some students in made many conclusions but not all were related to the purpose of the project.
- Suggestions for future analysis can also include the use of this study in related areas or improvements for further studies.

**Overall Presentation**

- There was a marked improvement in the correct use of spelling and grammar.
- Students used the correct mathematical jargon and symbols appropriately most of the time.

**Areas of Strength**

- Many of the submissions were of a high quality and related to real-world situations to which mathematical concepts were applied. A few were unique and interesting and represented the investigation of good ideas.
- There was evidence indicating that students applied a high degree of effort to the task. They understood the focus of their topic selection and conducted the requisite research thoroughly.

**Areas of Weakness**

- The CSEC Additional Mathematics rubric was absent from the samples submitted by some centres.
- Some SBA project titles and aims were not clear, not concise and not well defined.
- Teachers must be alert to any suspected plagiarism. Plagiarism must not be condoned and projects should be marked accordingly.
- The majority of students did not incorporate or suggest any future analysis in their submissions.
- Some students failed to adequately connect their finding(s)/conclusion(s) to their aim(s), their data collected and their analysis of the data.

## Comments Specifically for Project B

### Title — *what the project is investigating?*

- Many of the students gave a name for the project rather than a title.
- In most cases, the students related to real-world problems.

### Purpose of Project — *why this project/investigation is being done?*

- In many cases, the purpose was a replica of the title.
- Few students were specific enough about the purpose of the project
- The majority of the students was **not** able to identify the variables in the project properly.

### Method of data collection — *how the data is being collected*

- Most students were able to describe the data collection method clearly.
- It is expected that students mention the type of data collected (primary, secondary) and/or sampling used for example, random sampling, stratified sampling. This occurred only in a few instances.

### **Presentation of data**

- Most students had a minimum of one table or chart.
- Most students used the graphs, figures and tables appropriately in their presentation.

### **Mathematical Knowledge (Analysis of Data)**

- Mathematical concepts in many cases were not used appropriately from the Additional Mathematics syllabus
- In many cases, students used concepts and representations which reflected Form 2 level Mathematics. For example, some students used only pie charts and bar charts.
- In many instances, the analysis was attempted, but students were not able to use it in their discussions. For example, standard deviation was calculated but was not used in the discussion (there was no mention of spread or skewness and what they mean? in relation to the project).
- Students were not too clear on the two approaches to be used in the analysis.
- In many cases the two approaches used were of the same concept. For example, mean, mode and median are all measures of central tendencies.

### **Discussion of Findings/Conclusions**

- In some instances there was no statement of findings. However, most statements made were linked to the solution of the problems.
- Suggestions for future analysis can also include the use of this study in related areas or improvements for further studies.

### **Overall Presentation**

- There was a marked improvement in the correct use of spelling and grammar.
- Students used the correct mathematical jargon and symbols most of the time.

### Paper 032 — Alternative to SBA

This paper tested candidates' ability to:

- Determine the maximum profit for a given profit function which was quadratic.
- Represent real-world situations using quadratic equations and solve for unknown variables.
- Derive an expression, in algebraic form, for the area of a composite figure given its dimensions.
- Determine the range of values of an unknown side for which the area of the shape exists.

There were 83 candidates who wrote this paper, 81 of them attempted the question. The mean mark was 5.26 with a standard deviation of 4.46 compared with 8.69 and 7.75 respectively in 2014. No candidate received full marks (20 marks). The highest mark recorded was 18 out of 20.

This paper was very poorly done. Only about 3.7 per cent of candidates were able to achieve 75 per cent and above. Of the three parts, which comprised this paper, Part (b) was done the best. Candidates need to know the functions of calculus and when it should be used as opposed to using normal algebraic procedures.

In Part (a), candidates were required to use differentiation, or some other suitable method, to determine the maximum daily profit of a company. Many candidates tried to solve for  $x$  by factorizing the expression, completing the square or using the quadratic formula. Obviously, candidates did not connect differentiation with this question. After this part, the majority of candidates understood that substitution was necessary to determine the number of hundred dollars needed, even if the value they substituted was obtained erroneously. In Part (a), some of the candidates who solved the given equation obtained the values  $x = -30$  and  $x = 10$ . They then concluded that  $x = 10$  would give the maximum profit of \$1000.

#### Common Errors

Candidates solved for the roots of the equation rather than differentiate and equate to 0.

Candidates did not multiply by 100 to get the required dollar value for their answer.

In Part (b), candidates were required to determine dimensions of a square and rectangle given specific conditions. This part was relatively well done compared with the other two parts. Expressions for the dimensions of the square and rectangle were often correctly obtained, thus  $A = x^2 + (x+3)(x+6)$  was usually given as the total area although there were many variations presented such as  $w = s + 3$  and  $l = w + 3$ , where  $s$  represented the length of the square,  $w$  represented the width of the rectangle and  $l$  the length of the rectangle. The candidates who represented the total area correctly invariably obtained a quadratic equation and even though some factorized incorrectly, they were able to apply their answers to produce acceptable dimensions for each shape. Some candidates used the trial and error method to arrive at the correct dimensions. This method was not penalized once supporting statements were seen.

In Part (c) (i), candidates were required to obtain an expression for area in terms of  $x$  only. This part was poorly done and it was seen that candidates could not apply the formula for the area of a triangle to algebraic processes. Many knew they needed to add the areas of the rectangle and triangle, but were confused by what should be done next. Many candidates did not recognize the need to use Pythagoras' theorem in representing  $h$ , the height of the isosceles triangle in terms of its base,  $x$ , and its slant side which was 4 cm. In cases where Pythagoras' theorem was used, it was incorrectly applied.



In Part (c) (ii), candidates were required to find the required domain for area. Very few candidates even attempted this part and it was clear that they could not link area in Part (c) (i) to the length of a side,  $x$ , even though this was stated in the question. Additionally, candidates could not relate the need for  $x$  to be positive in order to represent length. Candidates also did not use the fact that the height,  $h$ , of the triangle had to be positive in order to find the required range for  $x$  using the result

$$h = \sqrt{\frac{64 - x^2}{4}} > 0 \text{ which would imply that } \sqrt{64 - x^2} > 0 .$$

With respect to candidates' responses on this question, it is recommended that candidates be given more practice questions. This will allow them to apply theory to real situations.

### Solutions

- (a) Maximum daily profit = \$10 000
- (b) (i) Dimensions of the square are 5.5 cm by 5.5 cm  
(ii) Dimensions of the rectangle are 11.5 cm by 8.5 cm
- (c) (i) Area of shaded surface =  $4x + \frac{x\sqrt{64 - x^2}}{4}$   
(ii) Domain of  $x$   $\{x : 0 < x < 8, x \in \mathbf{R}\}$

Generally, teachers need to ensure that the syllabus is completed with sufficient time to give students time to do practice exercise and past papers.