

Appendix D

Logarithms

THE LOGARITHM TO BASE 10 of a number is the exponent or power to which 10 must be raised to yield the number. Since 1000 is 10^3 , the logarithm to base 10 of 1000 (written $\log 1000$) is 3. Similarly, $\log 10000 = 4$, $\log 10 = 1$, $\log 0.1 = -1$, and $\log 0.001 = -3$.

Most hand calculators have a log key. When a number is entered into the calculator, its logarithm to base 10 can be found by pressing the log key. In this way we find that $\log 50 = 1.69897$ and $\log 0.035 = -1.45593$. Also, $\log 1 = 0$, which reflects the fact that $10^0 = 1$.

NATURAL LOGARITHMS are taken to the base $e = 2.718$, rather than 10. They can be found on most hand calculators by pressing the ln key. Since $e^0 = 1$, we have $\ln 1 = 0$.

Examples:

$$\begin{array}{ll} \log 971 = 2.9872 & \ln 971 = 6.8783 \\ \log 9.71 = 0.9872 & \ln 9.71 = 2.2732 \\ \log 0.0971 = -1.0128 & \ln 0.0971 = -2.3320 \end{array}$$

Exercises: Find the logarithms to base 10 of the following numbers.

- (a) 454 (f) 0.621
(b) 5280 (g) 0.9463
(c) 96500 (h) 0.0353
(d) 30.48 (i) 0.0022
(e) 1.057 (j) 0.0002645

- Ans. (a) 2.6571 (f) -0.2069
(b) 3.7226 (g) -0.02397
(c) 4.9845 (h) -1.4522
(d) 1.4840 (i) -2.6576
(e) 0.0241 (j) -3.5776

ANTILOGARITHMS: Suppose we have an equation such as $3.5 = 10^{0.544}$; then we know that 0.544 is the log to base 10 of 3.5. Or, inversely, we can say that 3.5 is the *antilogarithm* (or *inverse logarithm*) of 0.544. Finding the antilogarithm of a number is simple with most hand calculators: Simply enter the number; then press first the inverse key and then the log key. Or, if the base is e rather than 10, press the inverse and ln keys.

Exercises: Find the numbers corresponding to the following logarithms.

- (a) 3.1568 (f) 0.9142
(b) 1.6934 (g) 0.0008
(c) 5.6934 (h) -0.2493
(d) 2.5000 (i) -1.9965
(e) 2.0436 (j) -2.7994

- Ans. (a) 1435 (f) 8.208
 (b) 49.37 (g) 1.002
 (c) 4.937×10^5 (h) 0.563 2
 (d) 316.2 (i) 0.010 08
 (e) 110.6 (j) 0.001 587

BASIC PROPERTIES OF LOGARITHMS: Since logarithms are exponents, all properties of exponents are also properties of logarithms.

- (1) The logarithm of the product of two numbers is the sum of their logarithms. Thus,

$$\log ab = \log a + \log b \quad \log(5280 \times 48) = \log 5280 + \log 48$$

- (2) The logarithm of the quotient of two numbers is the logarithm of the numerator minus the logarithm of the denominator. For example,

$$\log \frac{a}{b} = \log a - \log b \quad \log \frac{536}{24.5} = \log 536 - \log 24.5$$

- (3) The logarithm of the n th power of a number is n times the logarithm of the number. Thus,

$$\log a^n = n \log a \quad \log(4.28)^3 = 3 \log 4.28$$

- (4) The logarithm of the n th root of a number is $1/n$ times the logarithm of the number. Thus,

$$\log \sqrt[n]{a} = \frac{1}{n} \log a \quad \log \sqrt{32} = \frac{1}{2} \log 32 \quad \log \sqrt[3]{792} = \frac{1}{3} \log 792$$

Solved Problem

- 1 Use a hand calculator to evaluate (a) $(5.2)^{0.4}$, (b) $(6.138)^3$, (c) $\sqrt[3]{5}$, (d) $(7.25 \times 10^{-11})^{0.25}$.

- (a) Enter 5.2; press y^x key; enter 0.4; press = key. The displayed answer is 1.934.
 (b) Enter 6.138; press y^x key; enter 3; press = key. The displayed answer is 231.2.
 (c) Enter 5; press y^x key; enter 0.333 3; press = key. The displayed answer is 1.710.
 (d) Enter 7.25×10^{-11} ; press y^x key; enter 0.25; press = key. The displayed answer is 2.918×10^{-3} .

Exercises

- 2 Evaluate each of the following.

- | | |
|---------------------------------------|--|
| (1) $28.32 \times 0.082 54$ | (5) $\frac{1}{239}$ |
| (2) $573 \times 6.96 \times 0.004 81$ | (6) $\frac{0.572 \times 31.8}{96.2}$ |
| (3) $\frac{79.28}{63.57}$ | (7) $47.5 \times \frac{779}{760} \times \frac{273}{300}$ |
| (4) $\frac{65.38}{225.2}$ | (8) $(8.642)^2$ |

(9) $(0.08642)^2$

(10) $(11.72)^3$

(11) $(0.0523)^3$

(12) $\sqrt{9463}$

(13) $\sqrt[3]{946.3}$

(14) $\sqrt{0.00661}$

(15) $\sqrt[3]{1.79}$

(16) $\sqrt[4]{0.182}$

(17) $\sqrt{643} \times (1.91)^3$

(18) $(8.73 \times 10^{-2})(7.49 \times 10^6)$

(19) $(3.8 \times 10^{-5})^2(1.9 \times 10^{-5})$

(20) $\frac{8.5 \times 10^{-45}}{1.6 \times 10^{-22}}$

(21) $\sqrt{2.54 \times 10^6}$

(22) $\sqrt{9.44 \times 10^5}$

(23) $\sqrt{7.2 \times 10^{-13}}$

(24) $\sqrt[3]{7.3 \times 10^{-14}}$

(25) $\sqrt{\frac{(1.1 \times 10^{-23})(6.8 \times 10^{-2})}{1.4 \times 10^{-24}}}$

(26) $2.04 \log 97.2$

(27) $37 \log 0.0298$

(28) $6.30 \log (2.95 \times 10^3)$

(29) $8.09 \log (5.68 \times 10^{-16})$

(30) $(2.00)^{0.714}$

<i>Ans.</i> (1) 2.337	(9) 0.007467	(17) 177	(25) 0.73
(2) 19.2	(10) 1611	(18) 6.54×10^5	(26) 4.05
(3) 1.247	(11) 0.000143	(19) 2.7×10^{-14}	(27) -56
(4) 0.2902	(12) 97.27	(20) 5.3×10^{-23}	(28) 21.9
(5) 0.00418	(13) 30.76	(21) 1.59×10^3	(29) -123
(6) 0.189	(14) 0.0813	(22) 9.72×10^2	(30) 1.64
(7) 44.3	(15) 1.21	(23) 8.5×10^{-7}	
(8) 74.67	(16) 0.653	(24) 4.2×10^{-5}	