Appendix C

Exponents

POWERS OF 10: The following is a partial list of powers of 10. (See also Appendix E.)

$$10^0 = 1$$
$$10^1 = 10$$
$$10^2 = 10 \times 10 = 100$$
$$10^3 = 10 \times 10 \times 10 = 1000$$
$$10^4 = 10 \times 10 \times 10 \times 10 = 10\,000$$
$$10^5 = 10 \times 10 \times 10 \times 10 \times 10 = 100\,000$$
$$10^6 = 10 \times 10 \times 10 \times 10 \times 10 \times 10 = 1\,000\,000$$

$$10^{-1} = \frac{1}{10} = 0.1$$
$$10^{-2} = \frac{1}{10^2} = \frac{1}{100} = 0.01$$
$$10^{-3} = \frac{1}{10^3} = \frac{1}{1000} = 0.001$$
$$10^{-4} = \frac{1}{10^4} = \frac{1}{10\,000} = 0.000\,1$$

In the expression 10^5, the *base* is 10 and the *exponent* is 5.

MULTIPLICATION AND DIVISION: In multiplication, exponents of like bases are added:

$$a^3 \times a^5 = a^{3+5} = a^8 \qquad 10^7 \times 10^{-3} = 10^{7-3} = 10^4$$
$$10^2 \times 10^3 = 10^{2+3} = 10^5 \qquad (4 \times 10^4)(2 \times 10^{-6}) = 8 \times 10^{4-6} = 8 \times 10^{-2}$$
$$10 \times 10 = 10^{1+1} = 10^2 \qquad (2 \times 10^5)(3 \times 10^{-2}) = 6 \times 10^{5-2} = 6 \times 10^3$$

In division, exponents of like bases are subtracted:

$$\frac{a^5}{a^3} = a^{5-3} = a^2 \qquad \frac{8 \times 10^2}{2 \times 10^{-6}} = \frac{8}{2} \times 10^{2+6} = 4 \times 10^8$$
$$\frac{10^2}{10^5} = 10^{2-5} = 10^{-3} \qquad \frac{5.6 \times 10^{-2}}{1.6 \times 10^4} = \frac{5.6}{1.6} \times 10^{-2-4} = 3.5 \times 10^{-6}$$

SCIENTIFIC NOTATION: Any number may be expressed as an integral power of 10, or as the product of two numbers one of which is an integral power of 10. For example,

$$2806 = 2.806 \times 10^3 \qquad 0.045\,4 = 4.54 \times 10^{-2}$$
$$22\,406 = 2.240\,6 \times 10^4 \qquad 0.000\,06 = 6 \times 10^{-5}$$
$$454 = 4.54 \times 10^2 \qquad 0.003\,06 = 3.06 \times 10^{-3}$$
$$0.454 = 4.54 \times 10^{-1} \qquad 0.000\,000\,5 = 5 \times 10^{-7}$$

OTHER OPERATIONS: A nonzero expression with an exponent of zero is equal to 1. Thus,

$$a^0 = 1 \qquad 10^0 = 1 \qquad (3 \times 10)^0 = 1 \qquad 8.2 \times 10^0 = 8.2$$

A power may be transferred from the numerator to the denominator of a fraction, or vice versa, by changing the sign of the exponent. For example,

$$10^{-4} = \frac{1}{10^4} \qquad 5 \times 10^{-3} = \frac{5}{10^3} \qquad \frac{7}{10^{-2}} = 7 \times 10^2 \qquad -5a^{-2} = -\frac{5}{a^2}$$

The meaning of the fractional exponent is illustrated by the following:

$$10^{2/3} = \sqrt[3]{10^2} \qquad 10^{3/2} = \sqrt{10^3} \qquad 10^{1/2} = \sqrt{10} \qquad 4^{3/2} = \sqrt{4^3} = \sqrt{64} = 8$$

To take a power to a power, multiply exponents:

$$(10^3)^2 = 10^{3\times 2} = 10^6 \qquad (10^{-2})^3 = 10^{-2\times 3} = 10^{-6} \qquad (a^3)^{-2} = a^{-6}$$

To extract the square root, divide the exponent by 2. If the exponent is an odd number it should first be increased or decreased by 1, and the coefficient adjusted accordingly. To extract the cube root, divide the exponent by 3. The coefficients are treated independently. Thus,

$$\sqrt{9\times 10^4} = 3\times 10^2 \qquad \sqrt{4.9\times 10^{-5}} = \sqrt{49\times 10^{-6}} = 7.0\times 10^{-3}$$

$$\sqrt{3.6\times 10^7} = \sqrt{36\times 10^6} = 6.0\times 10^3 \qquad \sqrt[3]{1.25\times 10^8} = \sqrt[3]{125\times 10^6} = 5.00\times 10^2$$

Most hand calculators give square roots directly. Cube roots and other roots are easily found using the y^x key.

Exercises

1 Express the following in powers of 10.

(*a*) 326 (*d*) 36 000 008 (*g*) 0.000 002 (*i*) $\sqrt{0.000\,081}$

(*b*) 32 608 (*e*) 0.831 (*h*) 0.000 706 (*j*) $\sqrt[3]{0.000\,027}$

(*c*) 1006 (*f*) 0.03

Ans. (*a*) 3.26×10^2 (*d*) $3.600\,000\,8\times 10^7$ (*g*) 2×10^{-6} (*i*) 9.0×10^{-3}
(*b*) $3.260\,8\times 10^4$ (*e*) 8.31×10^{-1} (*h*) 7.06×10^{-4} (*j*) 3.0×10^{-2}
(*c*) 1.006×10^3 (*f*) 3×10^{-2}

2 Evaluate the following and express the results in powers of 10.

(*a*) 1500×260 (*e*) $\dfrac{1.728\times 17.28}{0.000\,172\,8}$ (*i*) $(\sqrt[3]{2.7\times 10^7})(\sqrt[3]{1.25\times 10^{-4}})$

(*b*) $220\times 35\,000$ (*f*) $\dfrac{(16\,000)(0.000\,2)(1.2)}{(2000)(0.006)(0.000\,32)}$ (*j*) $(1\times 10^{-3})(2\times 10^5)^2$

(*c*) $40\div 20\,000$ (*g*) $\dfrac{0.004\times 32\,000\times 0.6}{6400\times 3000\times 0.08}$ (*k*) $\dfrac{(3\times 10^2)^3(2\times 10^{-5})^2}{3.6\times 10^{-8}}$

(*d*) $82\,800\div 0.12$ (*h*) $(\sqrt{14\,400})(\sqrt{0.000\,025})$ (*l*) $8(2\times 10^{-2})^{-3}$

Ans. (*a*) 3.90×10^5 (*e*) 1.728×10^5 (*i*) 1.5×10^1
(*b*) 7.70×10^6 (*f*) 1×10^3 (*j*) 4×10^7
(*c*) 2.0×10^{-3} (*g*) 5×10^{-5} (*k*) 3×10^5
(*d*) 6.9×10^5 (*h*) 6.0×10^{-1} (*l*) 1×10^6