Appendix B

Trigonometry Needed for College Physics

FUNCTIONS OF AN ACUTE ANGLE: The trigonometric functions most often used are the sine, cosine, and tangent. It is convenient to put the definitions of the functions of an acute angle in terms of the sides of a right triangle.

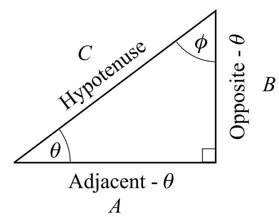
In any right triangle: The *sine* of either acute angle is equal to the length of the side opposite that angle divided by the length of the hypotenuse. The *cosine* of either acute angle is equal to the length of the side adjacent to that angle divided by the length of the hypotenuse. The *tangent* of either acute angle is equal to the length of the side opposite that angle divided by the length of the side adjacent to that angle.

If θ and ϕ are the acute angles of any right triangle and A, B, and C are the sides, as shown in the diagram, then

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{B}{C} \qquad \sin\phi = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{A}{C}$$

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{A}{C} \qquad \cos\phi = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{B}{C}$$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{B}{A} \qquad \tan\phi = \frac{\text{opposite}}{\text{adjacent}} = \frac{A}{B}$$



Note that $\sin\theta = \cos\phi$; thus the sine of any angle equals the cosine of its complementary angle. For example,

$$\sin 30^\circ = \cos(90^\circ - 30^\circ) = \cos 60^\circ \qquad \cos 50^\circ = \sin(90^\circ - 50^\circ) = \sin 40^\circ$$

As an angle increases from 0° to 90°, its sine increases from 0 to 1, its tangent increases from 0 to infinity, and its cosine decreases from 1 to 0.

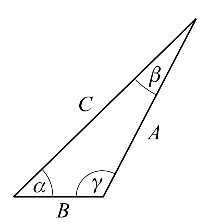
LAW OF SINES AND OF COSINES: These two laws give the relations between the sides and angles of *any* plane triangle. In any plane triangle with angles α, β, and γ and sides opposite A, B, and C, respectively, the following relations apply:

Law of Sines

$$\frac{A}{\sin\alpha} = \frac{B}{\sin\beta} = \frac{C}{\sin\gamma}$$

or

$$\frac{A}{B} = \frac{\sin\alpha}{\sin\beta} \quad \frac{B}{C} = \frac{\sin\beta}{\sin\gamma} \quad \frac{C}{A} = \frac{\sin\gamma}{\sin\alpha}$$



Law of Cosines

$$A^2 = B^2 + C^2 - 2BC\ \cos\alpha$$
$$B^2 = A^2 + C^2 - 2AC\ \cos\beta$$
$$C^2 = A^2 + B^2 - 2AB\ \cos\gamma$$

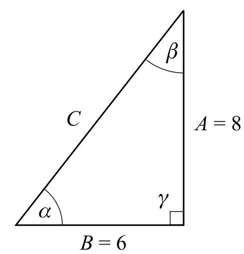
If the angle θ is between 90° and 180°, as in the case of angle C in the above diagram, then

$$\sin\theta = \sin(180^\circ - \theta) \qquad \text{and} \qquad \cos\theta = -\cos\ (180^\circ - \theta)$$

Thus
$$\sin 120^\circ = \sin(180^\circ - 120^\circ) = \sin 60^\circ = 0.866$$
$$\cos 120^\circ = -\cos(180^\circ - 120^\circ) = -\cos 60^\circ = -0.500$$

Solved Problems

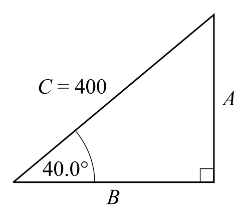
1 In right triangle ABC, given $A = 8$, $B = 6$, $\gamma = 90^\circ$. Find the values of the sine, cosine, and tangent of angle α and of angle β.



$$C = \sqrt{8.0^2 + 6.0^2} = \sqrt{100} = 10$$

$$\sin\alpha = A/C = 8.0/10 = 0.80 \qquad \sin\beta = B/C = 6.0/10 = 0.60$$
$$\cos\alpha = B/C = 6.0/10 = 0.60 \qquad \cos\beta = A/C = 8.0/10 = 0.80$$
$$\tan\alpha = A/B = 8.0/6.0 = 1.3 \qquad \tan\beta = B/A = 6.0/8.0 = 0.75$$

2 Given a right triangle with one acute angle 40.0° and hypotenuse 400. Find the other sides and angles.

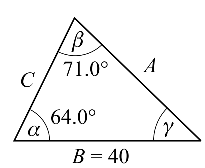


$$\sin 40.0^\circ = \frac{A}{400} \qquad \text{and} \qquad \cos 40.0^\circ = \frac{B}{400}$$

Using a calculator, we find that $\sin 40.0^\circ = 0.642\,8$ and $\cos 40.0^\circ = 0.766\,0$. Then

$$a = 400 \sin 40.0^\circ = 400(0.642\,8) = 257$$
$$b = 400 \cos 40.0^\circ = 400(0.766\,0) = 306$$
$$B = 90.0^\circ - 40.0^\circ = 50.0^\circ$$

3 Given triangle ABC with $\alpha = 64.0^\circ$, $\beta = 71.0^\circ$, $B = 40.0^\circ$. Find A and C.



$$\gamma = 180.0^\circ - (\alpha + \beta) = 180.0^\circ - (64.0^\circ + 71.0^\circ) = 45.0^\circ$$

By the law of sines,

$$\frac{A}{\sin\alpha} = \frac{B}{\sin\beta} \qquad \text{and} \qquad \frac{C}{\sin\gamma} = \frac{B}{\sin\beta}$$

so
$$A = \frac{B\sin\alpha}{\sin\beta} = \frac{40.0 \sin 64.0^\circ}{\sin 71.0^\circ} = \frac{40.0(0.898\,8)}{0.945\,5} = 38.0$$

and
$$C = \frac{B\sin\gamma}{\sin\beta} = \frac{40.0 \sin 45.0^\circ}{\sin 71.0^\circ} = \frac{40.0(0.707\,1)}{0.945\,5} = 29.9$$

4 (*a*) If $\cos\alpha = 0.438$, find α to the nearest degree. (*b*) If $\sin\beta = 0.800\,0$, find β to the nearest tenth of a degree. (*c*) If $\cos\gamma = 0.712\,0$, find γ to the nearest tenth of a degree.

(*a*) On your calculator use the inverse and cosine keys to get $\alpha = 64^\circ$; or if you have a $\cos^{-1}$ key use it.

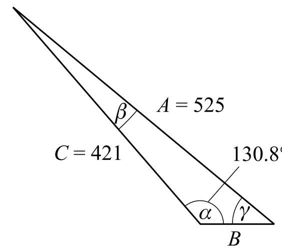
(*b*) Enter 0.800 0 on your calculator and use the inverse and sine keys to get $\beta = 53.1^\circ$.

(*c*) Use your calculator as in (*a*) to get 44.6°.

5 Given triangle ABC with $\alpha = 130.8^\circ$, $A = 525$, $C = 421$. Find B, β, and γ.

$$\sin 130.8^\circ = \sin(180^\circ - 130.8^\circ) = \sin 49.2^\circ = 0.757$$

Most hand calculators give $\sin 130.8^\circ$ directly.



For γ: $\sin\gamma = \dfrac{C\sin\alpha}{A} = \dfrac{421\sin 30.8^\circ}{525} = \dfrac{421(0.757)}{525} = 0.607$

from which $\gamma = 37.4^\circ$.

For β: $\beta = 180^\circ - (\gamma + \alpha) = 180^\circ - (37.4^\circ + 130.8^\circ) = 11.8^\circ$

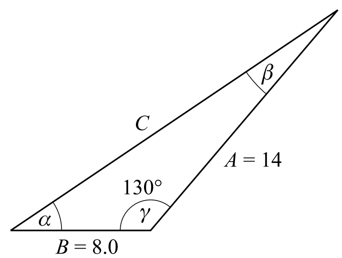
For B: $B = \dfrac{A\sin\beta}{\sin\alpha} = \dfrac{525\sin 11.8^\circ}{\sin 130.8^\circ} = \dfrac{525(0.204)}{0.757} = 142$

6 Given triangle ABC with $A = 14$, $B = 8.0$, $\gamma = 130^\circ$. Find C, α, and β.

$$\cos 130^\circ = -\cos(180^\circ - 130^\circ) = -\cos 50^\circ = -0.64$$

For C: By the law of cosines,

$$C^2 = A^2 + B^2 - 2AB\cos 130^\circ$$
$$= 14^2 + 8.0^2 - 2(14)(8.0)(-0.643) = 404$$



and $C = \sqrt{404} = 20$.

For α: By the law of sines,

$$\sin\alpha = \frac{A\sin\gamma}{C} = \frac{14(0.766)}{20.1} = 0.533$$

and $\alpha = 32^\circ$.

For β: $\beta = 180^\circ - (\alpha + \gamma) = 180^\circ - (32^\circ + 130^\circ) = 18^\circ$

Exercises

7 Solve each of the following right triangles ABC, with $\gamma = 90^\circ$.

(*a*) $\alpha = 23.3^\circ$, $C = 346$ (*d*) $A = 25.4$, $B = 38.2$
(*b*) $\beta = 49.2^\circ$, $B = 222$ (*e*) $B = 673$, $C = 888$
(*c*) $\alpha = 66.6^\circ$, $A = 113$

Ans. (*a*) $\beta = 66.7^\circ$, $A = 137$, $B = 318$ (*d*) $\alpha = 33.6^\circ$, $\beta = 56.4^\circ$, $C = 45.9$
(*b*) $\alpha = 40.8^\circ$, $A = 192$, $C = 293$ (*e*) $\alpha = 40.7^\circ$, $\beta = 49.3^\circ$, $A = 579$
(*c*) $\beta = 23.4^\circ$, $B = 48.9$, $C = 123$

8 Solve each of the following oblique triangles ABC.

(*a*) $A = 125$, $\alpha = 54.6^\circ$, $\beta = 65.2^\circ$ (*e*) $B = 50.4$, $C = 33.3$, $\beta = 118.5^\circ$
(*b*) $B = 321$, $\alpha = 75.3^\circ$, $\gamma = 38.5^\circ$ (*f*) $B = 120$, $C = 270$, $\alpha = 118.7^\circ$
(*c*) $B = 215$, $C = 150$, $\beta = 42.7^\circ$ (*g*) $A = 24.5$, $B = 18.6$, $C = 26.4$
(*d*) $A = 512$, $B = 426$, $\alpha = 48.8^\circ$ (*h*) $A = 6.34$, $B = 7.30$, $C = 9.98$

Ans. (*a*) $B = 139$, $C = 133$, $\gamma = 60.2^\circ$ (*e*) $A = 25.1$, $\alpha = 26.0^\circ$, $\gamma = 35.5^\circ$
(*b*) $A = 339$, $C = 218$, $\beta = 66.2^\circ$ (*f*) $A = 344$, $\beta = 17.8^\circ$, $\gamma = 43.5^\circ$
(*c*) $A = 300$, $\alpha = 109.1^\circ$, $\gamma = 28.2^\circ$ (*g*) $\alpha = 63.2^\circ$, $\beta = 42.7^\circ$, $\gamma = 74.1^\circ$
(*d*) $C = 680$, $\beta = 38.8^\circ$, $\gamma = 92.4^\circ$ (*h*) $\alpha = 39.3^\circ$, $\beta = 46.9^\circ$, $\gamma = 93.8^\circ$