R2S™ Nexus

Next Generation Knowledge Economy

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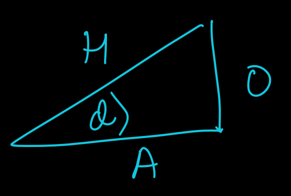
The 21st Century

TRIG DEFINITIONS & IDENTITIES



SOH CAH TOA — applies only to a ⊥ Δ

$\sin\theta = \dfrac{opp}{hyp} \Leftrightarrow \operatorname{cosec}\theta$ or $\csc\theta = \dfrac{hyp}{opp}$

$\cos\theta = \dfrac{adj}{hyp} \Leftrightarrow \sec\theta = \dfrac{hyp}{adj}$

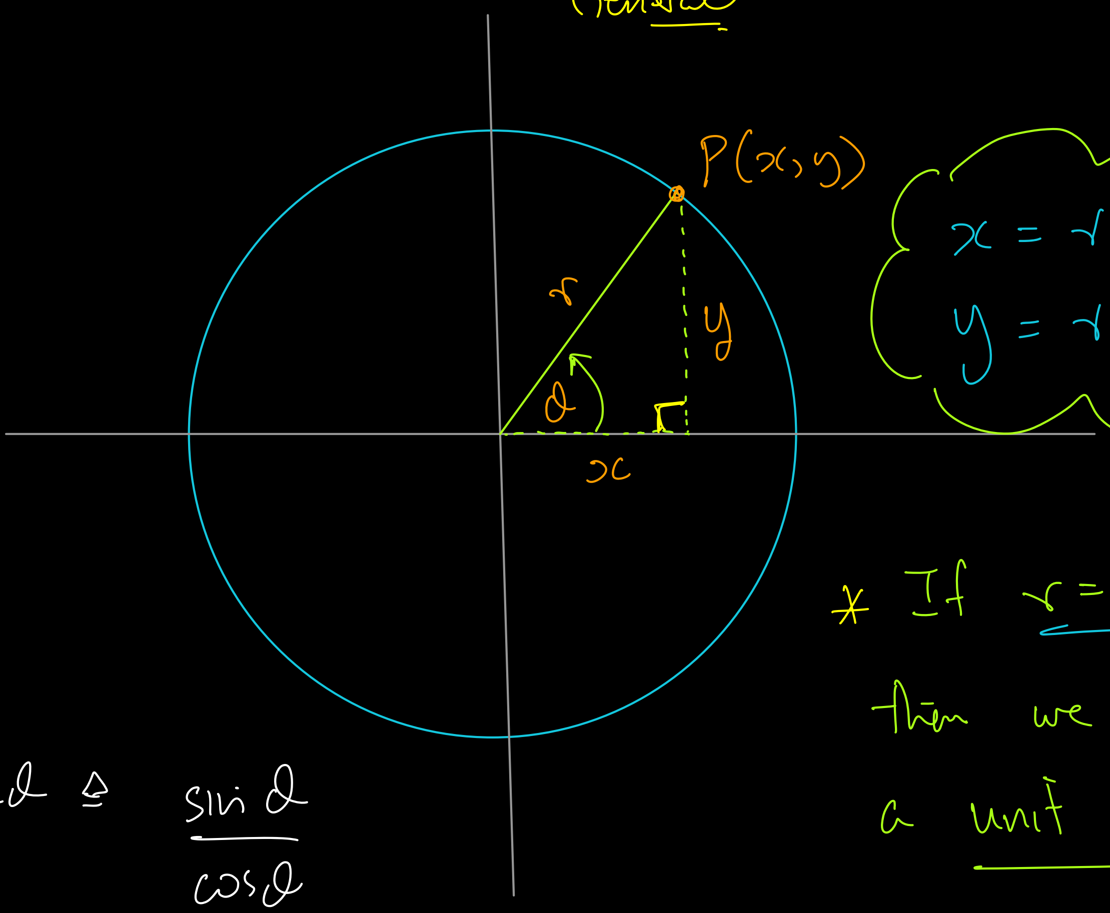
$\tan\theta = \dfrac{opp}{adj} \Leftrightarrow \operatorname{cotan}\theta$ or $\cot\theta = \dfrac{adj}{opp}$

INVERSE vs RECIPROCAL

$x = \sin^{-1}\theta$ $x = (\sin\theta)^{-1}$

$\Leftrightarrow \theta = \sin x$ $= \dfrac{1}{\sin\theta}$

General



$x = r\cos\theta$
$y = r\sin\theta$

* If $r = 1$ then we have a unit circle

$\tan\theta \triangleq \dfrac{\sin\theta}{\cos\theta}$

$\cot\theta \triangleq \dfrac{\cos\theta}{\sin\theta}$

IDENTITIES

A. Pythagoreans

1. $\sin^2\theta + \cos^2\theta \equiv 1$
2. $1 + \tan^2\theta \equiv \sec^2\theta$
3. $1 + \cot^2\theta \equiv \operatorname{cosec}^2\theta$

B. Compound Angle T.I.

1. $\sin(A \pm B) \equiv \sin A\cos B \pm \cos A\sin B$
2. $\cos(A \pm B) \equiv \cos A\cos B \mp \sin A\sin B$
3. $\tan(A \pm B) \equiv \dfrac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$

C. Double Angle T.I.s

1. $\sin 2A \equiv 2\sin A\cos A$
2. $\cos 2A \equiv \cos^2 A - \sin^2 A$
   $\equiv 2\cos^2 A - 1$
   $\equiv 1 - 2\sin^2 A$
3. $\tan 2A \equiv \dfrac{2\tan A}{1 - \tan^2 A}$

5. Prove the following identities:

5.1 $\sin A \tan A + \cos A = \sec A$

Proof: $\text{LHS} = \sin A \tan A + \cos A$ Reason

$= \sin A \dfrac{\sin A}{\cos A} + \cos A$

$= \dfrac{\sin^2 A}{\cos A} + \dfrac{\cos A}{1}$

$= \dfrac{\sin^2 A + \cos^2 A}{\cos A}$ but $\sin^2 A + \cos^2 A = 1$

$= \dfrac{1}{\cos A}$

$= \sec A$ (by defn)

$= \text{RHS}$ ∎

Show that 5.2 $(\cos A + \sin A)^2 + (\cos A - \sin A)^2 = 2$

PROOF: $\text{LHS} = (\cos A + \sin A)^2 + (\cos A - \sin A)^2$

$= \cos^2 A + \cancel{2\cos A \sin A} + \sin^2 A$
$+ \cos^2 A - \cancel{2\cos A \sin A} + \sin^2 A$

$= 2(\cos^2 A + \sin^2 A)$

$= 2$

$= \text{RHS}$ Q.E.D.

Show that 5.10 $\dfrac{1-\cos\phi}{\sin\phi} = \dfrac{1}{\csc\phi + \cot\phi}$

APPROACH I

PROOF:

$\dfrac{1-\cos\phi}{\sin\phi} = \dfrac{1}{\sin\phi} - \dfrac{\cos\phi}{\sin\phi}$

$= \left(\dfrac{\operatorname{cosec}\phi - \cot\phi}{1}\right)\left(\dfrac{\operatorname{cosec}\phi + \cot\phi}{\operatorname{cosec}\phi + \cot\phi}\right)$

$= \dfrac{\operatorname{cosec}^2\phi - \cot^2\phi}{\operatorname{cosec}\phi + \cot\phi}$

$= \dfrac{(1 + \cancel{\cot^2\phi}) - \cancel{\cot^2\phi}}{\operatorname{cosec}\phi + \cot\phi}$

$= \dfrac{1}{\operatorname{cosec}\phi + \cot\phi}$ (shown)

APPROACH II

PROOF:

$$\frac{1-\cos\phi}{\sin\phi} = \left(\frac{1-\cos\phi}{\sin\phi}\right)\left(\frac{1+\cos\phi}{1+\cos\phi}\right)$$

$$= \frac{1-\cos^2\phi}{\sin\phi\,(1+\cos\phi)}$$

$$= \frac{\sin^2\phi}{\sin\phi\,(1+\cos\phi)} \qquad \begin{matrix}\times \frac{1}{\sin^2\phi} \\ \times \frac{1}{\sin^2\phi}\end{matrix}$$

$$= \frac{1}{\frac{1}{\sin\phi}(1+\cos\phi)}$$

$$= \frac{1}{\operatorname{cosec}\phi + \cot\phi} \qquad \text{QED}$$

APPROACH III

PROOF:

$$\frac{1}{\operatorname{cosec}\phi + \cot\phi} = \left(\frac{1}{\frac{1}{\sin\phi} + \frac{\cos\phi}{\sin\phi}}\right)\left(\frac{\sin\phi}{\sin\phi}\right)$$

$$= \frac{\sin\phi}{1+\cos\phi}$$

$$= \left(\frac{\sin\phi}{1+\cos\phi}\right)\left(\frac{1-\cos\phi}{1-\cos\phi}\right)$$

$$= \frac{\sin\phi\,(1-\cos\phi)}{1-\cos^2\phi}$$

$$= \frac{\sin\phi\,(1-\cos\phi)}{\sin^2\phi}$$

$$= \frac{1-\cos\phi}{\sin\phi}$$

T.30 Simplify the following.

(i) $2\sin 3\theta\cos 3\theta$

(iii) $\cos^2 3\theta + \sin^2 3\theta$

(v) $\sin(\theta-\alpha)\cos\alpha + \cos(\theta-\alpha)\sin\alpha$

(vii) $\dfrac{\sin 2\theta}{2\sin\theta}$

(v) Recall: $\sin(A+B) \equiv \sin A\cos B + \cos A\sin B$

$\therefore$ when $A = \theta-\alpha$ & $B = \alpha$

then $A+B = (\theta-\alpha)+\alpha$
$= \theta$

$$\sin(\theta-\alpha)\cos\alpha + \cos(\theta-\alpha)\sin\alpha$$

$$= \sin\left[(\theta-\alpha)+\alpha\right]$$

$$= \sin\theta$$

(iv) $\dfrac{\sin 2\theta}{2\sin\theta} = \dfrac{\cancel{2}\,\cancel{\sin\theta}\cos\theta}{\cancel{2}\,\cancel{\sin\theta}}$

$= \cos\theta$ ✓

(ii) $\cos^2 3\theta - \sin^2 3\theta$ $= \cos 6\theta$ ✓

(iv) $1 - 2\sin^2\left(\dfrac{\theta}{2}\right)$ $= \cos\theta$ ✓

(vi) $3\sin\theta\cos\theta$

(viii) $\cos 2\theta - 2\cos^2\theta$

(vi) $3\sin\theta\cos\theta = \dfrac{3}{2}\left(2\sin\theta\cos\theta\right)$

$= \dfrac{3}{2}\sin 2\theta$ ✓

(viii) $\cos 2\theta - 2\cos^2\theta$

$\cos 2\theta - 2\cos^2\theta = 2\cos^2\theta - 1 - 2\cos^2\theta$

$= -1$ ✓

T.24 Solve the following equations for $0° \leqslant x \leqslant 360°$.

(i) $\operatorname{cosec} x = 1$	**(ii)** $\sec x = 2$	**(iii)** $\cot x = 4$
(iv) $\sec x = -3$	**(v)** $\cot x = -1$	**(vi)** $\operatorname{cosec} x = -2$

(vi) $\operatorname{cosec} x = -2$ take reciprocal of both sides

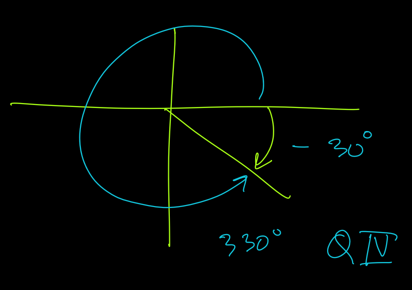
$\Leftrightarrow \sin x = -\dfrac{1}{2}$

S A
T C

$\therefore x = \sin^{-1}\left(-\dfrac{1}{2}\right)$ $\theta = 180° + \alpha$ $\theta = -\alpha$

$= -30°, \quad 180° + 30°$

$= 360° - 30°, \quad 210°$



$= 330°, \quad 210°$ ✓